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The RBF-Metamodel Development of Surface Eddy-Current Probe for the Surrogate Optimal Synthesis Problem

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Abstract
The eddy-current probe (EC-probe) RBF-metamodel is constructed, which can be used to eddy-current density calculate at control points of the space located in the control zone object surface, in the tasks of the EC-probe synthesis. As a coil of excitation of the EC-probe surface, an alternating current actuator, located above the control object with a constant specific electrical conductivity and magnetic penetration, was used in the work. The informatively model's verification was carried out by calculating the correlation coefficient multiplicity and verifying its statistical significance. When planning a multifactorial experiment, for the obtaining regression models purpose, LPτ-sequences were used, which proved to be promising for the surrogate optimization problems solution. The proposed computing technology has allowed developing an informative and adequate metamodel of the overhead EC-probe, which can be effectively used in optimal surrogate synthesis problems.

Keywords: eddy-current probe, eddy-current density distributions, metamodel, LPτ-sequence, MLP neural network

1. Introduction

For all types of eddy-current probe (EC-probe), a heterogeneous eddy-current distribution (ECD) is characteristic in an array of conductive material of the control object (CO). DEC is maximal in the superficial layer of the CO and decreases with distances from the coils of the EC-probe excitation along the surface, as well as in deeper layers of the CO.
The EC-probe signal essentially depends on the distribution of the ECD in the CO volume. Ideal for eddy-current control is the ECD homogeneous distribution in the EC-probe control zone, which completely minimizes the probe sensitivity dependence to the angle defect of its CO orientation.
It is technically impossible to realize such ECD distribution in classical EC-probe structures. At the same time, the coils developing EC-probe excitation in the certain structure system coils form with their concerted or counter-inclusion in the field allows the closest possible approximation to the ideal ECD distribution resulting [1].
Theoretically, the ideal ECD distribution-creating problem solution in the zone control is possible within the optimal synthesis problem framework. Such a task involves a multiple solution of the analysis EC-probe excitation structure problem, providing the ECD calculation in the set of points control zone located, both on the surface, and in deeper CO layers. The situation complexity involves the significant computing costs need and time resources for these calculations. Even for the analytic dependencies that simplest control cases exist, it is necessary to special functions and improper integrals calculate which greatly complicates the synthesis problem solution. For more complex control cases, for example, taking into account the EC-probe CO motion relative, analytic ECD expressions include multiple non-proper integrals. In sum, the synthesis problem solution is problematic as a resource constraints result.
The surrogate optimization problem can be solved, the use of which involves the surrogate model (metamodel) EC-probe developing. Under the metamodel understand the simple in the computer sense of the formal model to a more complex model, built on physical laws. This

approach allows us to solve optimal synthesis problems using the EC-probe metamodels, which are used when goal function formulating.
Purpose: the EC-probe RBF-metamodel developing, which can be used to ECD calculate at control space points located on the surface control zone object's, in the tasks of the EC-probe synthesis.

2. Specific instructions

As an excitation coil of the EC-probe overhead, an actuator with an alternating current I and a frequency ω that is located at a height z_0 over the control thickness object d with a constant specific electrical conductivity σ and a magnetic permeability μ_r (Fig. 1 a) was used in the work. The medium was considered linear, isotropic. The coil relative movement velocity to the object control is constant. The probe interaction with the control object is determined by the ratios obtained from the Maxwell equations. The current density components in coordinates x, y are respectively determined by the formulas:

$$J_x = \frac{1}{\mu_0 \cdot \mu_r} \cdot \left[\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right];$$
$$J_y = \frac{1}{\mu_0 \cdot \mu_r} \cdot \left[\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right]. \tag{1}$$

In this case, formulas [2] were used to induction components calculate with pre-defined partial derivatives - $\frac{\partial B_z}{\partial y}, \frac{\partial B_y}{\partial z}, \frac{\partial B_x}{\partial z}, \frac{\partial B_z}{\partial x}$:

$$B_x = \frac{\mu_0 \cdot \mu_r \cdot I}{8 \cdot \pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\xi}{\eta \cdot \left(1 - e^{2\cdot\gamma\cdot d}\right)} \cdot \left[\left\{ -\left(1+\lambda_0\right) \cdot e^{2\cdot\gamma\cdot d} + \nu_0 \cdot e^{\left(\gamma - \sqrt{\xi^2+\eta^2}\right)\cdot d} \right\} \cdot e^{\gamma\cdot z} + \right.$$
$$\left. + \left\{ 1 + \lambda_0 - \nu_0 \cdot e^{\left(\gamma - \sqrt{\xi^2+\eta^2}\right)\cdot d} \right\} \cdot e^{-\gamma\cdot z} \right] \cdot e^{-z_0 \cdot \sqrt{\xi^2+\eta^2}} \cdot S(\xi, \eta, a) \cdot e^{-j(x\cdot\xi + y\cdot\eta)} d\xi d\eta,$$

$$B_y = \frac{\mu_0 \cdot \mu_r \cdot I}{8 \cdot \pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\left(1 - e^{2\cdot\gamma\cdot d}\right)} \cdot \left[\left\{ -\left(1+\lambda_0\right) \cdot e^{2\cdot\gamma\cdot d} + \nu_0 \cdot e^{\left(\gamma - \sqrt{\xi^2+\eta^2}\right)\cdot d} \right\} \cdot e^{\gamma\cdot z} + \right.$$
$$\left. + \left\{ 1 + \lambda_0 - \nu_0 \cdot e^{\left(\gamma - \sqrt{\xi^2+\eta^2}\right)\cdot d} \right\} \cdot e^{-\gamma\cdot z} \right] \cdot e^{-z_0 \cdot \sqrt{\xi^2+\eta^2}} \cdot S(\xi, \eta, a) \cdot e^{-j(x\cdot\xi + y\cdot\eta)} d\xi d\eta,$$

$$B_z = j \cdot \frac{\mu_0 \cdot \mu_r \cdot I}{8 \cdot \pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\xi^2 + \eta^2}{\eta \cdot \gamma \cdot \left(1 - e^{2\cdot\gamma\cdot d}\right)} \cdot \left[\left\{ -\left(1+\lambda_0\right) \cdot e^{2\cdot\gamma\cdot d} + \nu_0 \cdot e^{\left(\gamma - \sqrt{\xi^2+\eta^2}\right)\cdot d} \right\} \cdot e^{\gamma\cdot z} - \right.$$
$$\left. - \left\{ 1 + \lambda_0 - \nu_0 \cdot e^{\left(\gamma - \sqrt{\xi^2+\eta^2}\right)\cdot d} \right\} \cdot e^{-\gamma\cdot z} \right] \cdot e^{-z_0 \cdot \sqrt{\xi^2+\eta^2}} \cdot S(\xi, \eta, a) \cdot e^{-j(x\cdot\xi + y\cdot\eta)} d\xi d\eta,$$

$$\gamma = \sqrt{\xi^2 + \eta^2 - j \cdot \sigma \cdot \mu_0 \cdot \mu_r \cdot \left(\upsilon_x \cdot \xi + \upsilon_y \cdot \eta\right) + j \cdot \omega \cdot \sigma \cdot \mu_0 \cdot \mu_r},$$

$$\lambda_0 = \frac{\left\{\gamma^2 - \mu_r^2 \cdot \left(\xi^2 + \eta^2\right)\right\} \cdot \left(1 - e^{-2\cdot\gamma\cdot d}\right)}{\left(\gamma + \mu_r \cdot \sqrt{\xi^2 + \eta^2}\right)^2 - \left(\gamma - \mu_r \cdot \sqrt{\xi^2 + \eta^2}\right)^2 \cdot e^{-2\cdot\gamma\cdot d}},$$

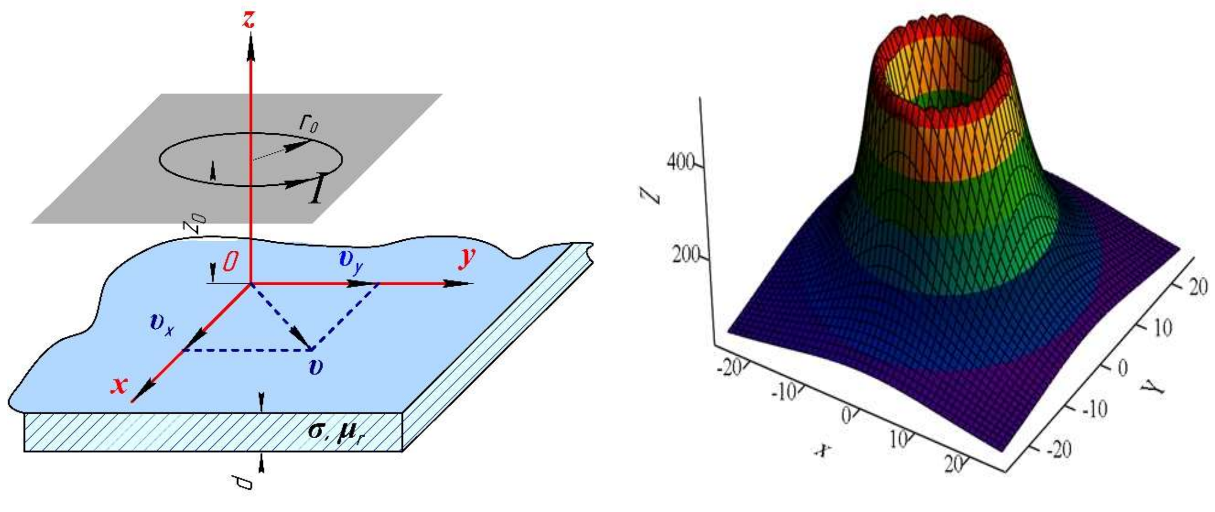
$$\nu_0 = \frac{4 \cdot \mu_r \cdot \gamma \cdot \sqrt{\xi^2 + \eta^2} \cdot e^{\left(\sqrt{\xi^2+\eta^2} - \gamma\right)\cdot d}}{\left(\gamma + \mu_r \cdot \sqrt{\xi^2 + \eta^2}\right)^2 - \left(\gamma - \mu_r \cdot \sqrt{\xi^2 + \eta^2}\right)^2 \cdot e^{-2\cdot\gamma\cdot d}},$$

where $S(\xi, \eta, a) = -j \cdot \frac{2 \cdot \pi \cdot a \cdot \eta}{\sqrt{\xi^2 + \eta^2}} \cdot J_1\left(a \cdot \sqrt{\xi^2 + \eta^2}\right)$ - coil shape function.

The metamodel development was carried out data using given in Table 1. Obtained target ECD function has the form shown in Fig.1 b.

Table 1. Output data for the ECD target function calculation

Conductive material thickness	d =5 mm
Leak location over the object control height	z_0 =2 mm
Frequency	f = 100 Hz
Radius turn	r_0 = 10 mm
Physical and technical material parameters	$\sigma = 2{,}5\cdot10^6$ S/m, $\mu_r = 1$
Current strength	I = 1 A
Along *x, y* coordinates velocity, respectively	υ_x = 0 m/s, υ_y = 0 m/s



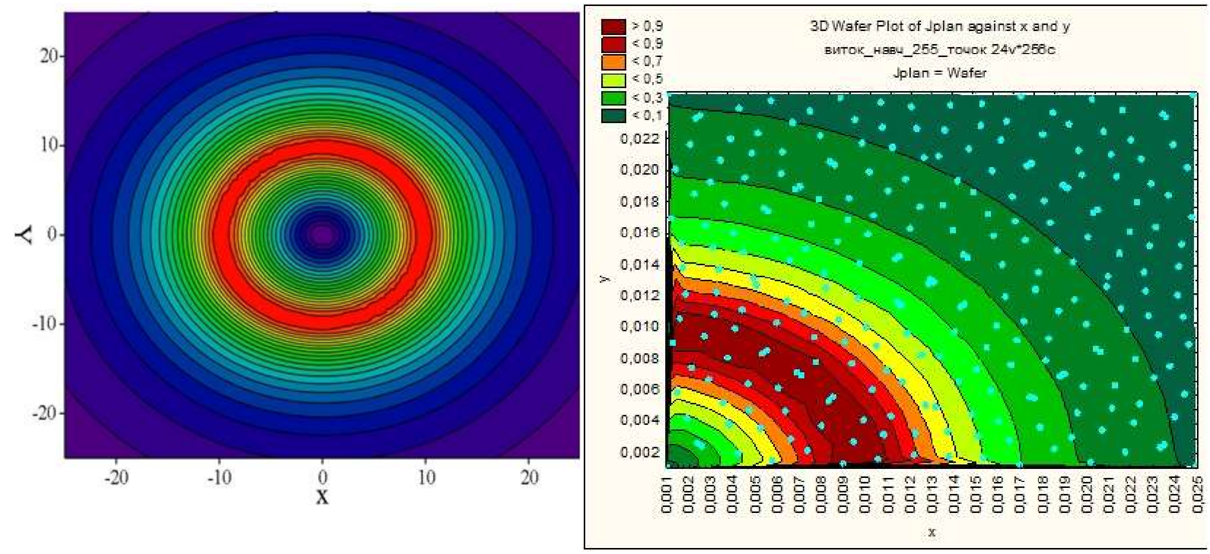
a) **b)**

Figure 1. Eddy-current probe, represented by a rotating coil of excitation coil:
a) geometric model; b) the exact target function in the form of the ECD distribution

The metamodel developing involves the solving three interrelated tasks: the computational experiment plan definition, the approximation model developing and the resulting metamodel adequacy and informatively verification.

Sequentially considered the each these separate tasks solution for the EC-probe metamodel developing task. Because of the potential response hypersurface topology complexity in this

study, it is advisable to use non-classical experiment planning methods, and computer methods to fill the multidimensional search space, which provide with a high probability homogeneous filling it with reference points, in which the resource-intensive target function values are subsequently calculated.
When experiment plan choosing among the possible options variety, the point generators that fill the search space and in the process of implementation of which the Sobol's LPτ-sequences are used should prevail. The following properties of this decision are the following properties of these sequences, noted in [3, 4]: the probability entering high of the probing sequence point in the search space in the vicinity of extremums points and bends of the target function response surface; main effects and factors interaction effects weakly correlated.
Therefore, the use of LPT-sequences in multifactorial experiment planning to obtain models regression is also solving surrogate optimization problems promising. Thus, to obtain the experimental plan points, LPτ-sequences (ζ_1, ζ_2) were used for N = 255. In the received probing points with corresponding coordinates (Fig. 2 b) the target function by the formula (1) values were calculated.



a) **b)**

Figure 2. Graphic representation of the goal function: a) line level target function; b) experiment plan N = 255, applied on the target function level line

The obtained ECD values at the plan points are used as the initial data for the second stage implementation - the metamodel developing. When problem solving, a heuristic method of meta-model developing based on neural networks, namely, a neural network on radial-basis functions is used.
To develop RBF-metamodels, an automated and user-defined strategy of random sampling developing with the sample is used in the following ratio: 70 % - educational, 15 % - control, 15 % - test. If necessary to improve the received metamodels parameters, these ratios changed by 80 %, 10 %, 10 %, respectively.
At the neural networks training stage, the best selection was carried out according to indicators: determination coefficient R^2, standard forecast error deviations ratio and data training *S.D.ratio*, average relative model error magnitude *MAPE,%*, residual average square error MS_R, histogram residue, diagram scatter. 348 neural networks were created for the N = 255 plan with the hidden neurons from 100 to 195 number, of which the best ones were selected (Table 2).

Table 2. RBF-metamodels

№ п/п	Metamodel	R^2 for educational, control, test sample	*S.D.ratio*	*MAPE,%*	MS_R
1	RBF-2-150-1(327)	0.999; 0,997; 0,992	0,0445	4	0,0001882
2	RBF-2-155-1(341)	0.999; 0,998; 0,992	0,03880	3,9	0,0001435
3	RBF-2-155-1(343)	0.999; 0,996; 0,992	0,04671	2,94	0,0002074
4	RBF-2-160-1(348)	0.999; 0,997; 0,993	0,03916	3,0	0,0001439
5	RBF-2-170-1(73)	0.999; 0,996; 0,995	0,0510	3,4	0,0002905
6	RBF-2-172-1(149)	0,999;0,996; 0,996	0,04252	5,9	0,000231

For the network with numbers 3, 6, respectively, in Fig. 3, 4 are indicators of the metamodel developing effectiveness and their parameters.

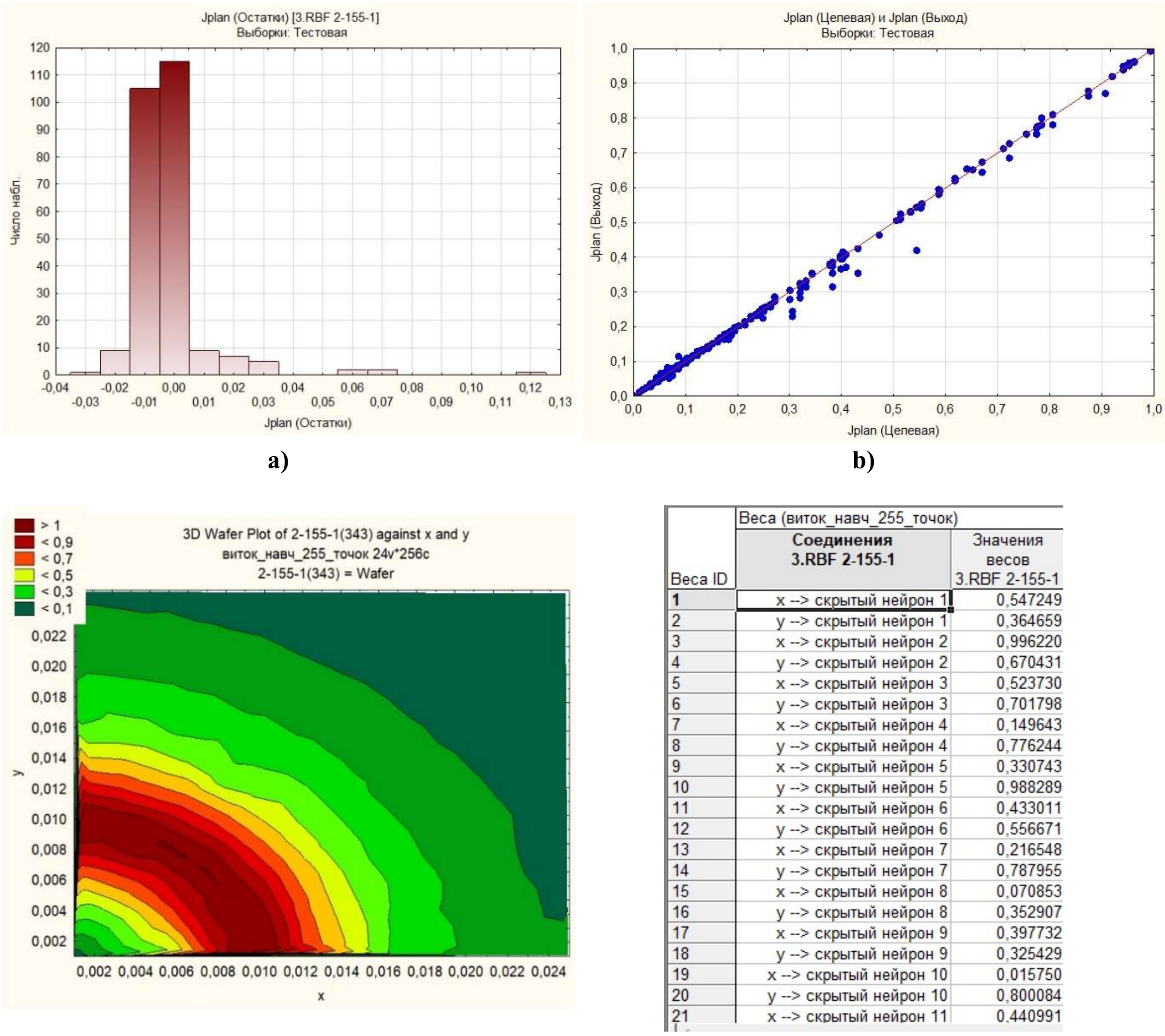
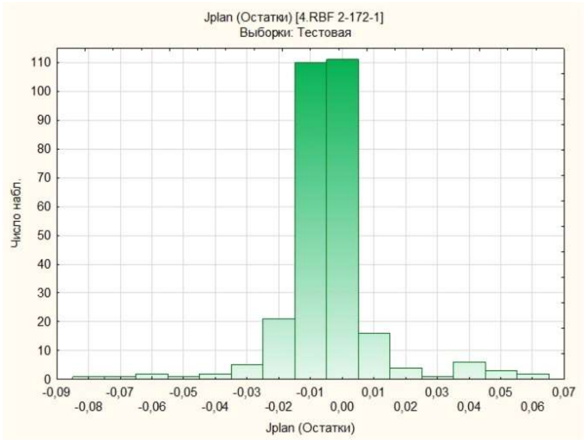
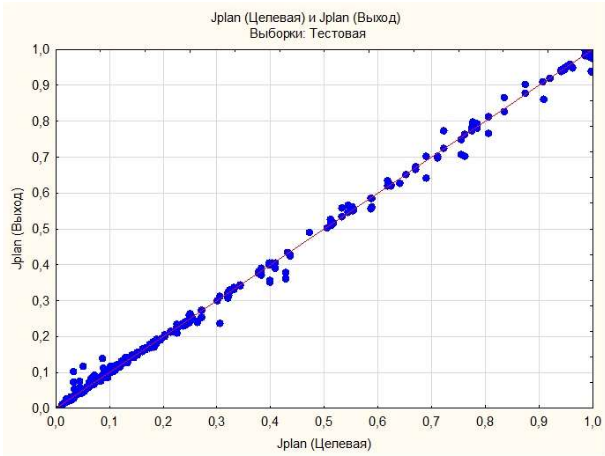


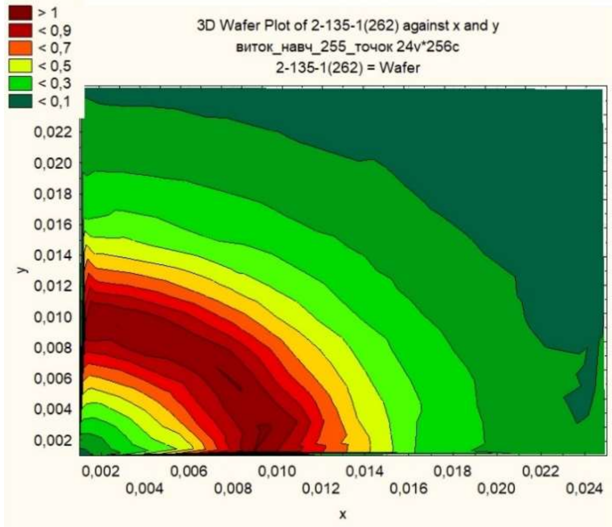
Figure 3. Neuronal network RBF-2-155-1 (343) (sample N = 255): a) residues histogram; b) diagram scattering of the target and approximation functions values; c) level line of the response surface reproduced at the training sample points; d) parameters value and weight created RBF-metamodel coefficients



a)



b)



c)

Веса ID	Веса (виток_навч_255_точок+контур+max) Соединения 4.RBF 2-172-1	Значения весов 4.RBF 2-172-1
340	y --> скрытый нейрон 170	0,980343
341	x --> скрытый нейрон 171	0,303192
342	y --> скрытый нейрон 171	0,482225
343	x --> скрытый нейрон 172	0,811004
344	y --> скрытый нейрон 172	0,682141
345	радиальный размах скрытый нейрон 1	0,010908
346	радиальный размах скрытый нейрон 2	0,055241
347	радиальный размах скрытый нейрон 3	0,038219
348	радиальный размах скрытый нейрон 4	0,012441
349	радиальный размах скрытый нейрон 5	0,069488
350	радиальный размах скрытый нейрон 6	0,058108
351	радиальный размах скрытый нейрон 7	0,067531
352	радиальный размах скрытый нейрон 8	0,011173
353	радиальный размах скрытый нейрон 9	0,070015
354	радиальный размах скрытый нейрон 10	0,059338
355	радиальный размах скрытый нейрон 11	0,033078
356	радиальный размах скрытый нейрон 12	0,033078
357	радиальный размах скрытый нейрон 13	0,042360
358	радиальный размах скрытый нейрон 14	0,059962
359	радиальный размах скрытый нейрон 15	0,073279
360	радиальный размах скрытый нейрон 16	0,060250
361	радиальный размах скрытый нейрон 17	0,055715

d)

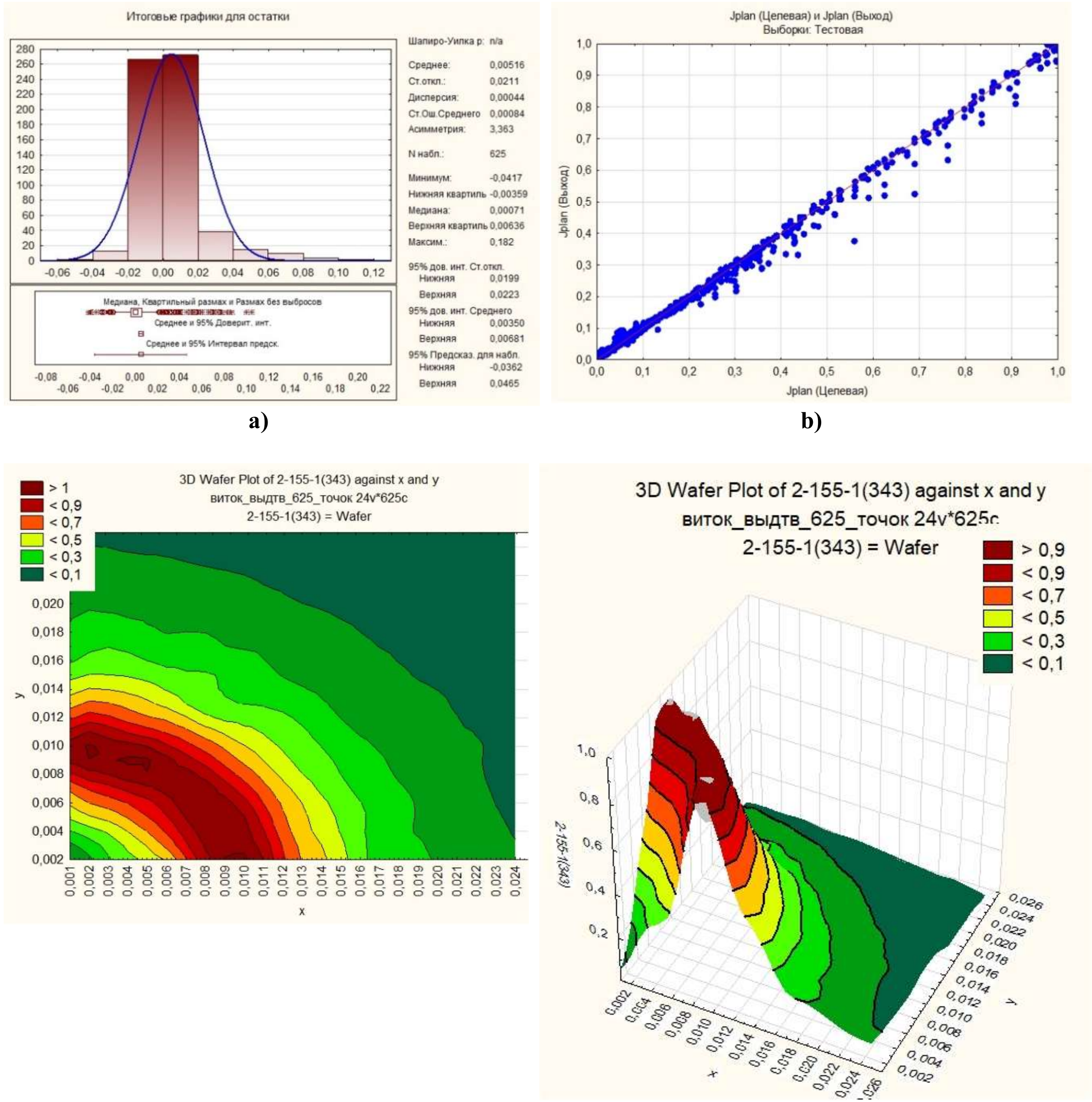
Figure 4. Neuronal network RBF-2-172-1 (149) (sample N = 255): a) residues histogram; b) diagram scattering of the target and approximation functions values o; c) level line of the response surface reproduced at the training sample points; d) parameters value and weight created RBF-metamodel coefficients

An important stage in meta-model developing is to check its adequacy. In the process of metamodel creating, multi-step validation is performed, the purpose of which is to control many numerical values obtained during its developing, including the neural network quality and the recovery assessment with its response surface using. Adequacy is usually established by checking the hypothesis F-criterion about the statistical adequacy dispersion insignificance σ^2_R and the reproducibility dispersion σ^2_D of the experiments results obtained by the mathematical model coefficients [3].

The model's informatively verification was carried out by correlation coefficient R multiplicity calculating and checking its statistical significance. The model is considered informatively $R^2 > 0,95$ and meaningful at the level $p \le 0,05$ of F-criterion (reliability $\ge 0,95$) significance.

The evaluation of the response surface recovery is made using the formula that describes the neural network output and is formed as a linear combination of the hidden layer neurons outputs with the obtained source neuron coefficients with the k-th neuron of the hidden layer w_k, the coordinates of the center k-th neuron $Cx_{1k}, Cx_{1k}, \ldots Cx_{lk}$, width k-th neuron [5]. Some values of the

weight coefficients are shown in Fig. 3, 4. Fig. 5, 6 shows the response surface restoration result obtained using the RBF-2-155-1 (343) and 2-172-1 (149) meta-models, which was performed in the whole range of variable $x \in [0; 25]$, $y \in [0; 25]$, with a step of 0.04, that is 625 points.
At the response surface reproduction stage, the received metamodel adequacy was evaluated according to the indicators: squares regression sum SS_D, remnants SS_R, total SS_T respectively; middle squares MS_D, MS_R, MS_T respectively; reproducibility dispersion σ^2_D, adequacy dispersion σ^2_R, general σ^2_T; reproducibility estimation standard error s_D, adequacy estimation standard error s_R, general s_T; determination coefficient R^2; standard deviations ratio $S.D.ratio$; average relative model error magnitude (or average error of approximation) $MAPE,\%$. For the created metamodels, these indicators are estimated and the results are summarized in Tables 3, 4.



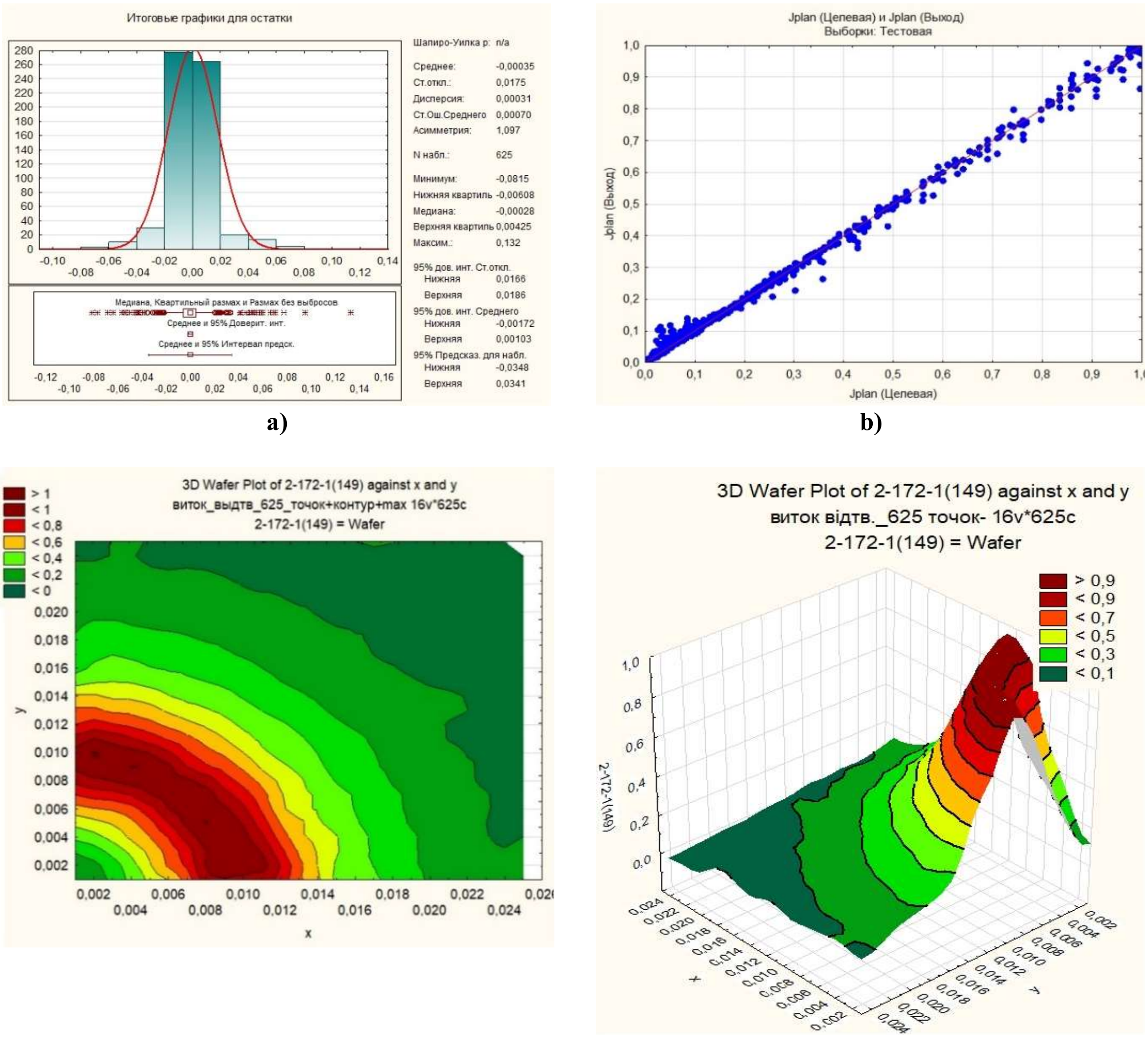
a) b)

c) d)

Figure 5. Reproduction of the response surface using the RBF-2-155-1 (343) metamodel: a) residues histogram; b) diagram target and restored function values dispersion; c) restored response surface level line; d) recovered response surface 3D-graph

Table 3. Checking RBF-2-155-1 (343) meta-model of the adequacy and informatively

N=625	Squares sum	Square average	Dispersion	Estimation standard error
Regression	$SS_D = 45{,}6625$	$MS_D = 22{,}8477$	$\sigma^2_D = 0{,}07317$	$s_D = 0{,}27051$
Remnants	$SS_R = 0{,}2933$	$MS_R = 0{,}000471$	$\sigma^2_R = 0{,}000471$	$s_R = 0{,}02171$
Total	$SS_T = 47{,}2325$	$MS_T = 0{,}07569$	$\sigma^2_T = 0{,}07569$	$s_T = 0{,}27512$
Criterion $F^{exp}_{vD;\, vR} > F^{crit}_{\alpha;\, vD;\, vR}$	$F^{exp}_{2;\, 622} = 48508;\ F^{crit}_{0{,}05;\, 2;\, 622} = 2{,}99873$			
Determination coefficient	$R^2 = 0{,}994342;\ F^{exp}_{2;\, 622} = 54655$			
Average error approximation	$MAPE = 7{,}31\ \%$			
Standard ratio deviations	$S.D.ratio = 0{,}0765$			



a) b) c) d)

Figure 6. Reproduction of the response surface by the metamodel RBF-2-172-1 (149): a) residues histogram; b) diagram target and restored function values dispersion; c) restored response surface level line; d) restored response surface 3D-graph

Table 4. Checking RBF-2-172-1 (149) meta-model of the adequacy and informatively

N=625	Squares sum	Square average	Dispersion	Estimation standard error
Regression	$SS_D = 45{,}6954$	$MS_D = 22{,}8477$	$\sigma^2_D = 0{,}07323$	$s_D = 0{,}27061$
Remnants	$SS_R = 0{,}1921$	$MS_R = 0{,}000308$	$\sigma^2_R = 0{,}000308$	$s_R = 0{,}017543$
Total	$SS_T = 47{,}2325$	$MS_T = 0{,}07569$	$\sigma^2_T = 0{,}07569$	$s_T = 0{,}275124$
Criterion $F^{exp}_{vD;\, vR} > F^{crit}_{\alpha;\, vD;\, vR}$	$F^{exp}_{2;\, 622} = 74180;\ F^{crit}_{0{,}05;\, 2;\, 622} = 2{,}99873$			
Determination coefficient	$R^2 = 0{,}99807;\ F^{exp}_{2;\, 622} = 160787$			
Average error approximation	$MAPE = 7{,}47\ \%$			
Standard ratio deviations	$S.D.ratio = 0{,}052$			

Thus, the proposed computational technology allowed developing an informative and adequate metamodel of the overhead eddy-current probe, which can be effectively used in optimal surrogate synthesis problems.

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