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Fuzzy economic order quantity model

The primary operation strategies and goals of most trade companies are to seek a high satisfaction to customer's demand and to become a maximum return on sales. To achieve these goals, the company must be able to effectively distribute current assets and minimize costs. The economic order quantity model is commonly used by decision maker as inventory control tool in real-world environment. It was tacitly assumed that the buyer must pay for the items purchased as soon as the items are received. Sometimes retailer can't pay at the given period just for economic reasons.

The economic order quantity model for deterioration items with two level of trade credit in one replenishment cycle, inspection errors, planned backorders, and sales returns in fuzzy environment is extends in this report.

Model formulation. Trade company, which temporarily does not have working capital, plans to replenish inventory by using of trade credit. The supplier offers the retailer a credit period of M-days. During this time, the retailer uses general revenue as investment resource with interest earned i_e. At the end of this period, the retailer pays off all units sold, keeping the rest for day-to-day expenses and stars paying for the interest charges on the unsold stock with the interest rate i_p. Basic interest rate i_p would be lower to i_w from the date of N, if the retailer pays off all units sold during M to N. The items received in a lot may contain defective items, and during screening process inspector may incorrectly classify a non-defective item as defective or incorrectly classify a defective item as non-defective. The items that are classified as defective and those returned by the consumers are kept in stock and sold at the end of the operation cycle. Additionally, the items stored in inventory face deterioration. Shortage is allowed. It is necessity determine the economic order quantity and length of operating cycle with the maximum revenue on sales.

It is known that D is the demand rate per unit time. Let us assume that λ is the replenishment rate per unit time. Then the inventory level of serviceable items at time t over the six periods in a cycle are determined by following differential equations:

$$\frac{I(t)}{dt} = \begin{cases} ((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda - D, & 0 \leqslant t < t_1, \\ ((1-\beta)(1-a_1)+\beta\cdot a_2)\lambda - D - \gamma\cdot\theta\cdot I, & t_1 \leqslant t < t_2, \\ -D-\gamma\cdot\theta\cdot I, & t_2 \leqslant t < t_3, \\ ((1-\beta_r)(1-a_{r1})+\beta_r\cdot a_{r2})\lambda_r - D - \gamma\cdot\theta\cdot I, & t_3 \leqslant t < t_4, \\ -D-\gamma\cdot\theta\cdot I, & t_4 \leqslant t < t_5, \\ \dfrac{-D}{1+\delta(t-t_5)}, & t_5 \leqslant t < T \end{cases} \tag{1}$$

with the boundary conditions $I(0) = I_b$, $I(t_1) = 0$, $I(t_2) = I_m$, $I(t_3) = I_s$, $I(t_4) = I_r$, $I(t_5) = 0$, $I(T) = I_b$, where β – the probability that the item is defective, a_1 – the probability of classifying a non-defective item as defective, a_2 – the probability of classifying a defective item as non-defective, λ_r – replenishment rate of items obtained for settle a quality claims per unit time, β_r – the probability that the item obtained for settle a quality claims is defective, a_{r1} – the probability of classifying a non-defective item obtained for settle a quality claims as defective, a_{r2} – the probability of classifying a defective item obtained for settle a quality claims as non-defective, θ – percentage of items deteriorated per unit time, γ – percentage of deteriorated items screened out from the inventory, I_b – shortage level, I_m – maximum serviceable inventory level, I_s – inventory level of serviceable items, I_r – inventory level of reworked items, T – length of the cycle.

It can be deduced from final solutions of (1) at $I(0) = I_b$, $I(t_1) = 0$, $I(t_2) = I_m$, $I(t_3) = I_s$, $I(t_4) = I_r$, $I(t_5) = 0$, $I(T) = I_b$ that

$$t_1 = \frac{I_b}{D - \lambda\left((1-\beta)(1-a_1)+\beta\cdot a_2\right)} \tag{2}$$

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$$t_2 = t_1 - \frac{1}{\gamma \cdot \theta} \ln\left(1 - \frac{\gamma \cdot \theta \cdot I_m}{((1-\beta)(1-a_1) + \beta \cdot a_2)\lambda - D}\right) \tag{3}$$

$$t_3 = t_2 - \frac{1}{\gamma \cdot \theta} \ln\left(\frac{I_s \cdot \gamma \cdot \theta + D}{I_m \cdot \gamma \cdot \theta + D}\right) \tag{4}$$

$$t_4 = t_3 - \frac{1}{\gamma \cdot \theta} \ln\left(\frac{I_r \cdot \gamma \cdot \theta - (((1-\beta_r)(1-a_{r1}) + \beta_r \cdot a_{r2})\lambda_r - D)}{I_s \cdot \gamma \cdot \theta - (((1-\beta_r)(1-a_{r1}) + \beta_r \cdot a_{r2})\lambda_r - D)}\right) \tag{5}$$

$$t_5 = t_4 - \frac{1}{\gamma \cdot \theta} \ln\left(\frac{D}{I_r \cdot \gamma \cdot \theta + D}\right) \tag{6}$$

$$T = t_5 - \frac{1}{\delta}\left(e^{\frac{-I_b \cdot \delta}{D}} - 1\right). \tag{7}$$

We may also deduce that economic order quantity is

$$Q = \lambda \cdot t_2. \tag{8}$$

In this paper, we have considered the demand rate, selling prices, interest paid rate, interest earned rate as fuzzy variables and thereby the return on sales becomes fuzzy variables on the credibility space $(X, P(X), Cr)$. If the decision maker wants to determine optimal pricing and inventory police such that fuzzy expected value of the return on sales is maximal, a fuzzy expected value model can be constructed as follows

$$E\left[ROS(I_b, I_m, I_s)\right] = E\left[\frac{\widetilde{R}_s - K - \widetilde{P}_c - \widetilde{C}_{si} - \widetilde{C}_{rg} - \widetilde{C}_h - \widetilde{C}_d - \widetilde{C}_s - \widetilde{C}_{un} - \widetilde{C}_{adi} - \widetilde{C}_{rnd} - \widetilde{IP} - \widetilde{IE}}{\widetilde{R}_s}\right] \tag{9}$$

subject to the constraints

$E_{min} \leqslant E[\widetilde{P}_c] \leqslant E_{max}$, investment amount on total production cost have an upper and lower limits;

$b \cdot c \cdot d \cdot \lambda(E[\widetilde{t_1}] + E[\widetilde{t_2}]) \leqslant W$, warehouse space where the items are to be stored is limitation;

$E[\widetilde{C}_h] < E[\widetilde{P}_c]$, holding cost cannot be more than total production cost;

$P\left(a_2 \leqslant 1 - \frac{u+1}{\beta \cdot \lambda(E[\widetilde{t_1}] + E[\widetilde{t_2}])}\right) \leqslant p_s$, providing good service of the customers,

where $\widetilde{R}_s$ – sale revenue, K – the ordering cost, $\widetilde{P}_c$ – the purchase cost, $\widetilde{C}_{si}$ – the (inspection) screening cost, $\widetilde{C}_{rg}$ – the cost for return the rejection items to supplier, $\widetilde{C}_h$ – the inventory holding cost, $\widetilde{C}_d$ – the deterioration cost, $\widetilde{C}_s$ – the shortage cost due to backlog, $\widetilde{C}_{un}$ – the opportunity cost due to lost sale, $\widetilde{C}_{adi}$ – the cost of accepting a defective items, $\widetilde{C}_{rnd}$ – the cost of rejection a non-defective items, $\widetilde{IP}$ – interest paid, $\widetilde{IE}$ – interest earned, b, c, d – overall dimension of production unit, W – capacity of warehouse, u – the maximum allowable number of defective items in the lot, p_s – level of direct consumer service.

The expected value model can be solved with generic approaches. Here in order to solve the expected value model, we choose one of heuristic algorithms inspired from evolution of nature such as Evolutionary Technologies of Directed Optimization [1] as the foundation to design an algorithm which integrates fuzzy simulation and Evolutionary Technologies of Directed Optimization, where the fuzzy simulation is employed to estimate the maximal revenue on sales, and Evolutionary Technologies of Directed Optimization is used to find the optimal solution.

Література. **1.** Snytyuk V. E. Compositional overcoming uncertainty in nonlinear multivariable optimization problems [Text] / V. E. Snytyuk //Artificial Intelligence. – 2004. – №4. – C. 207–210.