Algorithms for Identification of Stationary Transducers

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Abstract: **The possibility of identifying the parameters of the transducers using the integral Volterra equations of the second type is shown. It is suggested that the use of integrated dynamic models of transducers in some cases produces a more precise result than traditional methods based on the application of differential equations. The examples show that the proposed method can be used effectively to solve the tasks for parametric identification of transducers and can be the basis for building of software for application programs of general use and specialized evaluators.**

Keywords — transducers; parametr identification; integral dynamic models; quarter phase algorithms

I. Introduction

Any prior information can be used to solve the tasks for identification of the transducers (for example, transducer is linear, transition processes in it are of monotonous character, non-linearities are smooth; the structure of the model is known, and the numerical values of its parameters are unknown; the model structure and approximate values of its parameters, and so on, are known, etc. The methods of solving tasks for building models differ in how the characteristics of a dynamic object are presented (in a temporary or frequency area); the method of experimentation; methods of recovering unknown transducer parameters (least square methods, statistical decision theory, stochastic approximation, etc).

II. Setting objective of research

In this work, the tasks of building dynamic transducers are explored. It is assumed that the values of input *f*(*t*) and output *y*(*t*) signals are measured at time moments of t_i

$$0 \le t_1 \le t_2 \le \cdots \le t_N \le T, \quad (1)$$

with some errors

$$\tilde{f}(t_i) = f(t_i) + \delta_i, \max_{0\le i\le N}|\delta_i|, \quad (2)$$

$$\tilde{y}(t_i) = y(t_i) + \varepsilon_i, \max_{0\le i\le N}|\varepsilon_i| = \varepsilon, \quad (3)$$

where $\tilde{f}$ and $\tilde{y}$ are approximate values, *f* and *y* are precise signal values.

The traditional approach in solving such tasks for stationary transducers requires the application of ordinary differential equations of the type

$$\sum_{i=0}^{r} q_i\, y^{(i)}(t) = f(t), t \in [0,T], \quad (4)$$

where q_i are constant coefficients, that are determined from (1) − (5).

Among the main difficulties encountered in this approach is the incorrectness of the numerical differentiation of the experimentally set functions of $\tilde{f}$ and $\tilde{y}$in (2) and (3), if the model is to be replicated directly (4). Indirect search procedures are usually used to define q_i

III. PRESENTATION OF THE MAIN MATERIAL

In line with the developing integrated approach in this work, we will consider integrated dynamic models of the type [1, 2] which are equivalent to models (4),

$$y(t) + \int_0^t K(t,\tau)u(\tau)d\tau = F(t), \quad (5)$$

where $K(t,\tau)$ is to be determined value, $y(t)$ is an output signal, $F(t) := F(f;t)$ is a known function determined through the value of input signal.

Values $K(t,\tau)$ and $F(t)$ are determined from (5) according to statements:

$$K(t,\tau) = \sum_{i=1}^{m} q_i \frac{(t-\tau)^{i-1}}{(i-1)!},\ m \in N;$$

$$F(t) = \int_0^t \frac{(t-\tau)^{m-1}}{(m-1)!} f(\tau)ds + \sum_{i=1}^{m-1} C_i \frac{t^i}{i!} + \sum_{i=1}^{m-1} q_j \sum_{k=0}^{m-i-1} C_k \frac{t^{k+i}}{(k+i)!}, \quad (6)$$

where C_k are known values; q_i are indentified parameters; $f(t)$, $y(t)$ are input and output signals accordingly.

It is obvious that, in general, if you choose to build the transducer model, you can expect the model (5) to be more efficient than the model (4), because it allows the usage of direct methods of realization of operations on the left side of the equation. While the use of indirect (optimization) calculations is required to determine the values of q_i in (4). Perhaps these two approaches can complement each other and exercise mutual control in the sense of modeling accuracy. The practical task solving shows that for a sufficiently broad transducer class, the application of integrated dynamic models of the form (5) and, in particular, models equivalent to models (4), provides a basis for building robust numerical algorithms for the calculation of parameters of dynamic transducer models.

To form a system of linear algebraic equations with respect to unknown coefficients of q_i, $i=\overline{1,m}$ let's convert equation (5) taking into account (6) to the form

$$\sum_{i=1}^{m} q_i \left[\int_0^i \frac{(t-\tau)}{(i-1)!} y(\tau)d\tau - \sum_{k=0}^{m-i-1} C_k \frac{t^{k+i}}{(k+i)!}\right] = \qquad (7)$$

$$= \int_0^t \frac{(t-\tau)}{(m-1)!} F(\tau)d\tau + \sum_{j=0}^{m-1} C_k \frac{t_i}{i!} - y(t).$$

Hence for the fixing points (measuring) $t_j(j=\overline{0,N})$, where N is the number of fixations in the period of measurements T , $(t_N=T)$; $h=\frac{T}{N}$ is the measurement step, we obtain a system of linear algebraic equations relative to unknown coefficients.

$$Aq=F, \qquad (8)$$

Where $q=(q_1,\dots,q_m)^T, b=(b_1,\dots,b_m)^T$, and also:

$$A_{ji} = \int_0^{t_j} \frac{(t_j-\tau)^{i-1}}{(i-1)!} y(\tau)d\tau - \sum_{k=0}^{m-i-1} C_k \frac{t_j^{k+i}}{(k+i)!}, \qquad (9)$$

$$F_j = \int_0^t \frac{(t_j-\tau)^{m-1}}{(m-1)!} f(\tau)ds - y(t_j) + \sum_{i=0}^{m-1} C_i \frac{t_j^i}{i!}. \qquad (10)$$

It is obvious that, in general, if you choose to build the transducer model, you can expect the model (5) to be more efficient than the model (4), because it allows direct methods to implement operations on the left side of the equation. Whereas for determining values of $q_i(t)$ in (4) requires the use of indirect (optimization) computations. These two approaches are likely to complement each other and to exercise mutual control in the sense of modeling accuracy. The practical tasks show that for a sufficiently broad transducer class, the application of integrated models of the form (5) and, in particular, models equivalent to models (4), provides a basis for building robust numerical algorithms for the calculation of parameters of dynamic transducer models.

The methodic discussed in this article is based on the use of a trapezoid formula, the virtue of which is the simplicity of implementation and the high stability of computational algorithms.

The overall scheme for the implementation of the quadrature algorithm for the acquisition of integrated transducer models was discussed above, which specifies the way in which the system of linear algebraic equations is formed with respect to unknown parameters with matrix of left side of the form (9) and the right side of the form (10).

В выражения (9) и (10) входят интегралы:

Expression (9) and (10) contains integrals:

$$B_{ji} = \int_0^{t_j} \frac{(t_j-\tau)^{i-1}}{(i-1)!} y(\tau)d\tau,\ R_j = \int_0^{t_j} \frac{(t_j-\tau)^{m-1}}{(m-1)!} f(\tau)d\tau. \qquad (11)$$

Using Newton binomial for presenting expressions $(t_j-\tau)^{i-1}, (t_j-\tau)^{m-1}$ we get:

$$B_{ji} = \int_0^{t_j} \frac{(t_j-\tau)^{i-1}}{(n-1)!} f(\tau)d\tau = \frac{1}{(m-1)!} \sum_{l=0}^{i-1} C_{i-1}^l t^{i-1-l} (-1)^l \times \int_0^{t_j} s^l y(\tau)d\tau, \qquad (12)$$

$$R_j = \int_0^{t_j} \frac{(t_j-\tau)^{m-1}}{(n-1)!} f(\tau)d\tau = \frac{1}{(n-1)!} \sum_{k=0}^{m-1} C_n^k t_j^{m-1-k} (-1)^k \times \int_0^{t_j} s^k y(\tau)d\tau, \qquad (13)$$

where $C_n^m = \frac{n!}{m!(n-m)!}$.

Enter notations:

$$D_{ji} = \int_0^{t_j} \tau^l f(s)ds, Q_{jk} = \int_0^{t_j} \tau^k f(\tau)d\tau, \qquad (14)$$

$$\left(j=\overline{1,m},\ l=\overline{0,i-1},\ k=\overline{0,n-1}\right).$$

Using for calculating integrals D_{jl}, Q_{jk} trapezoid formula we get:

$$D_{jk} = \int_0^{t_j} s^l y(s)ds = \left[(jR)^l y(jR) + 2\sum_{p=1}^{j-1} (ph)^l y(ph)\right] h,$$

$$Q_{jk} = \int_0^{t_j} s^k f(s)ds =$$

$$= \left[(jh)^k f(jh) + 2\sum_{p=1}^{j-1} (ph)^k f(ph)\right] h. \qquad (15)$$

Then we get calculation expressions for B_{ji}, R_j:

$$B_{ji} \simeq \frac{1}{(i-1)!} \sum_{l=0}^{i-1} C_{i-1}^l t_j^{i-1-l} \left[(jh)^l y(jh) + 2\sum_{p=1}^{j-1} (ph)^l y(ph)\right] h, \qquad (16)$$

$$R_j \simeq \frac{1}{(n-1)!} \sum_{k=0}^{n-1} C_{n-1}^k t_j^{n-1-k} \left[(jh)^k f(jh) + 2\sum_{p=1}^{j-1} (ph)^k f(ph)\right] h. \qquad (17)$$

The elements of the system matrix of linear equations (8) and the right side are formed according to the expressions:

$$A_{ji} = B_{ji} - \sum_{k=0}^{m-i-1} C_k \frac{t_j^{k+i}}{(k+i)!}, \quad (18)$$

$$F_j = R_j - y(t_j) + \sum_{i=0}^{m-1} C_i \frac{t_j^i}{i!}. \quad (19)$$

So, the technique of the integral method of calculating model parameters (4.5) with the use of a trapezoid formula is as follows.

The source data is set to the numerical values of the input $f(t)$ and output signal $y(t)$ at points $t = t_j, j = \overline{1, m}$. The following operations are performed sequentially:

1. Calculate integrals (14) using quadrature formulas.

2. Calculation of B_{ji}, R_j by equations (16), (17).

3. Formation of the system matrix (8) According to the formula (18).

4. Formation of the right side of the system by formula (19).

5. System solution(8).

IV. CONCLUSION

Aiming to test performance and effectiveness of the given methodology and the developed set of programs, let's consider the model examples. We believe that the input and output signals are analytical and, in addition, the initial conditions and order of the model are known. The initial data is shown in table 1.

TABLE I

№	$F(t)$	$y(t)$	C_0	C_1	C_2	C_3	C_4	C_5
1	$3\sin t + 2\cos t$	$\sin t$	0	1	-	-	-	-
2	$e^{-t}\cos(10t - 11) + e^{-t}\sin(9t - 2)$	$te^{-t}\cos t$	0	1	2	-	-	-

Computational experiments were performed to test the methodology based on the algorithm proposed. The calculation results for examples 1 and 2 from Table 4.2 $(m = 2,\ h = 0{,}01,\ N = 250)$ and $(m = 3,\ h = 0{,}01,\ N = 250)$ are shown in tables 2 and 3, respectively.

TABLE 2

	Parameters		Initial data	
	q_1	q_2	C_0	C_1
Precise meanings	2	4	0	1
Obtained without initial data input	2,000069	3,99997	$2{,}55\cdot 10^{-16}$	0,99998
Obtained with initial data input	2,000033	3,99999	—	—

TABLE 3

	Parameters			Initial data		
	q_1	q_2	q_3	C_0	C_1	C_2
Precise meanings	4	– 3	5	0	1	2
Obtained without initial data input	4,0000099	–3,00004	5,0002	$3{,}178\cdot 10^{-4}$	0,9981654	–1,99142

From the results, it can be concluded that the algorithms reviewed are sufficiently effective in terms of accuracy and stability to errors of experimental data.

And can be the basis for building software for universal packages of application programmes and specialized control units.

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