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MATHEMATICAL MODELS OF ROAD CONSTRUCTION, RECONSTRUCTION AND REPAIR UNDER CONDITIONS OF UNCERTAINTY

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Abstract. The article highlights the importance of mathematical modeling in road construction, reconstruction, and repair in Ukraine. We can outline the importance of this research by taking into account such factors as financial instability and financial uncertainty, as well as the importance to comply with safety standards during any road-related maintenance work. We would also like to point out that there is an increasing need for new tools for optimization and improvement of construction operations. This need exists both due to factors of uncertainty, such as climatic anomalies, dynamics of road traffic, and technological progress in construction, as well as due to numerous social challenges that result in an increase in traffic load, for instance, an ongoing military conflict.

The main goal of our research is to implement mathematical modeling into different phases of any road-related maintenance work, from planning to executing, under conditions of limited finances. Achieving the aforementioned goal will allow to improve assessing and repairing the critical areas and will also improve the choice of optimal nondestructive inspection methods for identifying any damage in road coating and infrastructure. To achieve this goal, we use methods of statistical and system analysis, multi-criteria optimization, and decision-making theory. The main emphasis is on the development of strategic management decisions and their effectiveness.

We have managed to prove that the use of mathematical modeling during the planning phase of any road works allows us to effectively distribute the available financial resources even if they are limited. This practice also makes it possible to dynamically alter the volumes of funding depending on expert analysis of the importance or project completion status.

Upon completion of the initial planning stage, which includes repairing all critical areas of the road in order to minimize the risk of road traffic accidents and using methods of increased safety, we have managed to develop a plan with concrete steps. Following the aforementioned steps allows us to reduce a total number of road traffic accidents occurring in critical areas by 1.4-1.6 times. These results are possible to achieve without going over the established financial restrictions.

We have also developed criteria for assessing the quality of damage identification on the road coating. These criteria are based on the defect detection rate. We have provided a detailed explanation of how this rate can increase from 0,891 to 0,967.

Further scientific research conducted by the author of this paper will focus on a detailed analysis of the results of the implementation of illustrated mathematical models, especially as mathematical support of the analytical and information system for monitoring the state of road infrastructure.

Keywords: mathematical model, motor transport, roadway, critical area, nondestructive inspection method, condition of uncertainty.

Introduction

Modern world is in a state of constant development, which results in an expansion of roadways between cities, countries, and continents (Global Trends 2030, 2012). Apart from planning new road networks, it is also very important to reconstruct and repair the existing ones (Disclaimer: The Ukraine Rapid Damage and Needs Assessment, 2022). However, development and maintenance of road infrastructure demand not only considerable financial

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investments but also creation and improvement of project management methods, as well as methods for planning and forecasting such projects (Muhammad *et al*., 2021; Willar *et al*., 2023; Browning, 2018).

In Ukraine, mathematical modeling is very important in road construction, reconstruction, and repair, especially under conditions of unstable and uncertain funding, as well as under conditions of following special safety procedures. Climatic anomalies, dynamics of road traffic, technological progress in construction, as well as numerous social challenges causing an increase in traffic load due to the ongoing military conflict, make it necessary to develop new methods and tools for optimization of road construction (Zolotukhin *et al*., 2021).

However, a rapid development of mathematical modeling in road construction has become very noticeable in the motor transport industry throughout the years (Belyaev, 2020). Scientists work on improving methods of mathematic forecasting of the demand for transport services, create models to analyze the influence of uncertainty on the cost and duration of construction, and utilize artificial intelligence to analyze data to improve resource distribution and project management (Kazeem *et al*., 2023; Adesola *et al*., 2023).

The main purpose of this paper is to analyze mathematical modeling during the planning phase of construction, reconstruction, and repair of roads and their support structures under conditions of uncertainty and reduced funding. This analysis can improve the procedure of dealing with critical areas and help choose optimal methods of nondestructive testing to hasten the identification of damage to road coating and analyze the quality of road infrastructure components.

We have identified the following tasks required to reach the aforementioned goal:

– improvement of mathematical model for planning of construction, reconstruction, and repair of roads and related infrastructure under conditions of uncertainty and reduced funding;

– development of a plan for dealing with critical areas on roads;

– optimization of methods of nondestructive testing to hasten the identification of damage to road coating and analyzing the quality of road infrastructure components;

The scientific novelty of our study is as follows:

– we have improved mathematical models designed for establishing an effective functioning of modern road construction industry. These models are based on the models of expert analysis factoring in the uncertainty related to the volumes of funding. Using such models allows for improvement of the existing funding mechanism of construction objects by arranging them according to their cost and priority;

– we have improved the mechanisms for solving applied problems related to optimization of planning of road construction, reconstruction, and repair based on algorithmic diagrams of genetic algorithms and method of time series analysis. The mechanisms themselves are illustrated by the models of discrete optimization and can hasten the identification of damage. They are also used as criteria for choosing methods of nondestructive testing of road coating.

Analysis of literature

Below, we will analyze the results of the latest research and their practical application in road construction. Additionally, we will highlight the opportunities for further research related to the topic in question. This analysis is important to support the development and improve the quality of traffic infrastructure of Ukraine, both of which are important for social and economic development of Ukraine.

Mathematical models and methods used to solve problems of decision theory are not only the foundation of problems analyzed in this paper but a powerful mathematical tool. The

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development of modern computing devices and information technologies allows us to use them as programs for support and maintenance.

A group of scientists led by S. S. Kyzyma (Scientific work of Department, 2023) from the National University of Transport has made a noticeable contribution by developing a method for planning of reconstruction and repair of road coating based on technical and economic analysis. Renowned scientists, such as N. Sokolova (Kanin *et al*., 2022) and I. Sokolova (Markelova *et al*., 2022), have dedicated their research to mathematical methods used for planning and optimizing the operational use of roads under conditions of uncertainty. Among the scientists who have worked with the mathematical maintenance models are V. Babakov and A. Kravchenko (Kharchenko *et al*., 2023).

The following Ukrainian scientists also conduct their research in the field of road construction: professor of Dnipro State Technical University Borys Sereda, who studies mathematical modeling of transport and production systems (Sereda *et al*., 2023); professor of the National University of Transport Artur Onyshchenko, who analyzes mathematical models and methods for optimization of planning and managing road construction projects (Onyshchenko et al., 2019); professor of Vinnytsia National Technical University Serhii Pavlov, who researches methods to optimize the quality and speed of road construction (Wójcik *et al*., 2021). The following foreign scientists have dedicated their research to the problem in question, including Dr. Carlos Daganzo (Spain), who investigates the optimization of transport systems, including road networks, and is also a creator of several mathematical models for optimizing road traffic and infrastructure (Itani *et al*., 2021); professor of McGill University, Dr. Michael Florian, who describes methods of optimization of road networks and transport systems (Babazadeh *et al*., 2021); Henrik Fredriksson (2021) from Blekinge Institute of Technology, who studies mathematical modeling of transport systems for improving the decision-making and analysis of transporting-related issues.

Methods and materials

Creation of a mathematical model for planning of construction, reconstruction, and repair of roads and infrastructure under conditions of uncertainty and reduced financing

In general, planning construction, reconstruction, and repair of roads used for different purposes, as well as the whole related infrastructure, can be characterized by the necessity to define the tasks required to be completed during the given period and the calculation of required financial resources. The primary obstacle capable of influencing efficiency is the uncertainty of volumes of funding for construction programs. For instance, according to (On the report on the implementation of the Law of Ukraine), the volume of underfunding for intermediate-level maintenance and overhaul maintenance of public roads, as well as building new road infrastructure and provision of new gear, materials, etc., was 71.3% of the demand in 2022. According to (Markuts *et al*., 2023), general financing of roadbuilding will decrease by 14,9 billion UAH in 2023 as compared to the previous year. The main focus of this financing has also shifted from building new roads to repairing the ones destroyed as a result of the ongoing military conflict (Markuts *et al*., 2023).

One of the most common methods of addressing an issue of uncertainty related to funding is putting on hold all construction work on certain planned objects. Defining such objects is done by factoring in different criteria, for example, importance, current state, remaining cost of construction work, etc. Effective solution to this task requires a clear definition and a reasonable algorithm approach since there are a lot of different types of objects, as well as criteria for their evaluation and restriction.

Expert analysis can be used to evaluate the importance of each object based on different criteria. For example, social and economic importance of an object can be evaluated based on

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the following definitions: "high-priority, important, with average priority, low-priority". These definitions are then transformed into normalized scores from 0 to 1.

We can illustrate all criteria used to make decisions regarding the possible reduction of funding of road construction, reconstruction, and repair projects under conditions of underfunding as a hierarchical structure as shown in Fig. 1.

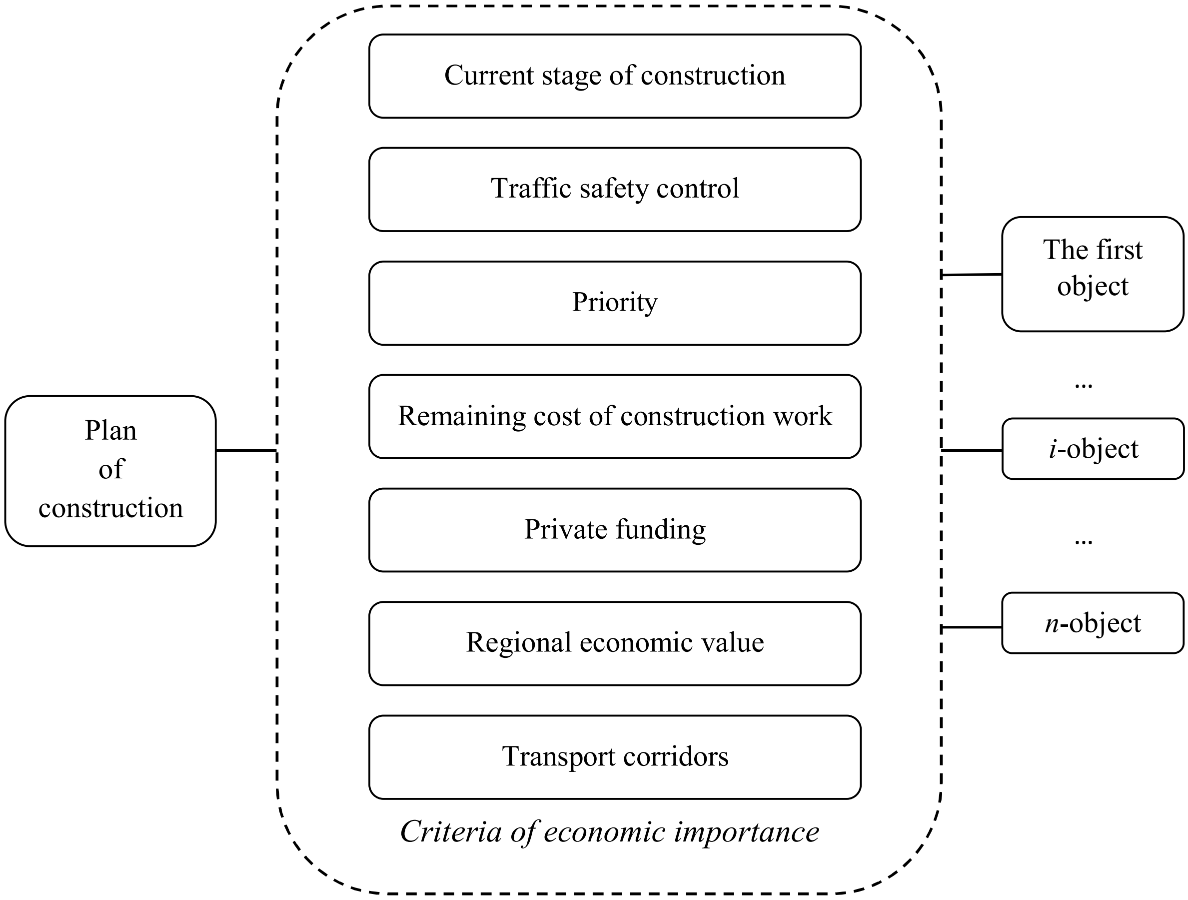

Figure 1. Simplified scheme of criteria of economic importance

According to data from Fig. 1, decision-making is affected by different criteria, such as criteria of economic importance. These criteria include:
- current stage of construction;
- traffic safety and dealing with critical areas;
- priority of construction, which is defined by the government;
- remaining cost of construction work;
- funding received from other sources (except the State Budget of Ukraine);
- evaluation of economic influence on the region;
- contribution to the development of international transport corridors and influence on traffic flow in general.

The aforementioned scheme from Fig. 1 indicates the types of work required to be completed on the roads of different types, including their support structures, In addition, it shows what types of work require funding.

Next, we use paired-comparison method to define the weight numbers of these criteria. We use the same method to analyze hierarchical structures (Mi *et al.*, 2022).

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Formulation of the problem. We will now analyze linear convolution for the previously chosen criteria according to the defined weight numbers $\rho_1, \rho_2, \ldots, \rho_m$:

$$\sum_{i \in I} \sum_{k=1}^{m} \rho_k v_{ik} z_i ,$$

where v_{ik} is defined as the value of i-object according to k-criterium, z_i is defined as the current stage of constructing i-object.

Correspondingly, the current stage of construction of each object depends on how the program is implemented:

$$z_i = f_i (x_{i1}, x_{i2}, \ldots, x_{ih^*}).$$

In this case, boolean value x_{ih} equals 1 if the h-stage of construction of the i-object is properly funded. If not, then $x_{ih} = 0$.

Function f_i has the following attributes:

- f_i X [0, 1];

The following condition $f_i (x^1_{i1}, x^1_{i2}, \ldots, x^1_{ih^*}) \geq f_i (x^2_{i1}, x^2_{i2}, \ldots, x^2_{ih^*})$ is valid for any boolean vector $(x^1_{i1}, x^1_{i2}, \ldots, x^1_{ih^*})$ greater than $(x^2_{i1}, x^2_{i2}, \ldots, x^2_{ih^*})$.

Solution algorithm. Statement of the problem where each stage is completed sequentially (the next stage cannot be completed unless all the previous ones are completed as well) can be illustrated in the following way:

$$x_{ih} \geq x_{ih+1}, h = 1, 2, \ldots, (h^* - 1), i = 1, 2, \ldots, n.$$

Below we provide a detailed illustration

$$X = \begin{pmatrix} x_{11} & x_{12} & \ldots & x_{1h^*} \\ x_{21} & x_{22} & \ldots & x_{2h^*} \\ \ldots & \ldots & \ldots & \ldots \\ x_{n1} & x_{n2} & \ldots & x_{nh^*} \end{pmatrix}.$$

Cost of all planned construction works:

$$B(X) = \sum_{i \in I} \sum_{h=1}^{h^*} b_{ih} x_{ih} ,$$

where b_{ih} is defined as the cost of all planned construction works on the h-stage.

If the real funding can be described as the value $B_0 < B$ (1), then the following matrix $X = (x_{ih})_{i\text{X}I,\, h = 1, 2, \ldots, h^*}$ is used, whereas:

$$X^* = \arg\max_{x \in X} \sum_{i \in I} \sum_{k=1}^{m} \rho_k v_{ik} f_i (x_{i1}, x_{i2}, \ldots, x_{ih^*}) \quad (1)$$

under conditions:

$$x_{ih} \geq x_{ih+1}, h = 1, 2, \ldots, (h^* - 1), \quad (2)$$

$$B(X) = \sum_{i \in I} \sum_{h=1}^{h^*} b_{ih} x_{ih} \geq B_0 . \quad (3)$$

Under conditions of uncertainty related to volumes of funding, the following problem can appear: find optimal matrixes $X_1^*, X_2^*, \ldots, X_d^*$ for the defined $B_1^0, B_2^0, \ldots, B_d^0$ ($B_1^0 < B_2^0 < \ldots < B_d^0$) values. The matrixes are required to meet the additional condition of a «build-up»:

$$X_1^* < X_2^* < \ldots < X_d^*.$$

Meeting the aforementioned condition allows us to slowly «build up» the plans for funding.

Formulation of the inverse problem. Now, we rewrite the previous problem (1) – (3) in the following way:

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$$\sum_{i=1}^{n}\sum_{k=1}^{m}\rho_k v_{ik} f_i\left(x_{i1},\, x_{i2},\, \ldots,\, x_{ih^*}\right)\to \max \tag{4}$$

$$\sum_{i=1}^{n}\sum_{h=1}^{h^*} b_{ih}x_{ih} \le B_0\,, \tag{5}$$

$$x_{ih} \ge x_{ih+1},\ h = 1, 2, \ldots, (h^* - 1),\ i = 1, 2, \ldots, n, \tag{6}$$

$$x_{ih} \text{X}\{0, 1\},\ h = 1, 2, \ldots, h^*,\ i = 1, 2, \ldots, n. \tag{7}$$

Here we also introduce new boolean values y_{ih}. These values are related to the previous x_{ih} values in the following way: $y_{ih} = 1$, if $x_{ih} = 1$ for $\forall k \le h$. In other cases, $y_{ih} = 0$, if $x_{ik} = 0$ for $\forall k > h$. We will use function $q(\cdot)$ to connect the values $x_{ih} = q(h, y_{i1}, y_{i2}, \ldots, y_{ih^*})$, $i = 1, 2, \ldots, n$, $h = 1, 2, \ldots, h^*$.

In this scenario, we can use a maximization problem equivalent to (4) – (7):

$$\sum_{i=1}^{n}\sum_{k=1}^{m}\rho_k v_{ik} z_i\left(y_{i1},\, y_{i2},\, \ldots,\, y_{ih^*}\right); \tag{8}$$

under conditions:

$$\sum_{h=1}^{h^*} y_{ih} = 1,\ i = 1, 2, \ldots, n, \tag{9}$$

$$\sum_{i=1}^{n}\sum_{h=1}^{h^*} \tilde{b}_{ih} y_{ih} \le B_0\,, \tag{10}$$

where $z_i(y_{i1}, y_{i2}, \ldots, y_{ih^*}) = f_i(q(1, y_{i1}, y_{i2}, \ldots, y_{ih^*}), \ldots, q(h^*, y_{i1}, y_{i2}, \ldots, y_{ih^*}))$, $i = 1, 2, \ldots, n$,

$$\tilde{b}_{ip} = \sum_{h=1}^{p} b_{ih}\ ,\ p = 1, \ldots, h^*.$$

Problems (8) – (10) are problems of linear programming with boolean values. A solution to this problem can be acquired by building a minorant in nondecreasing order. This minorant will correspond with objective function (8), as well as with rows of h^*-value boolean vectors, which correspond with condition (9) (Chen *et al.*, 2008).

Development of a plan for dealing with critical areas on roads

Formulation of the problem. Suppose the traffic flow consists of vehicles, which are divided into categories $k \in K = \{1, 2, \ldots, k^*\}$. They include light motor vehicles of different types and sizes, cargo vehicles of different types, sizes and capacities, buses, two-wheeled vehicles, etc. Then we divide the road into sectors g_i, $i \in I = \{1, 2, \ldots, n\}$. In this case, $p_{ik}(x_i)$ is defined as probability of traffic accident in sector g_i for k-category of vehicles under the condition of using measure $x_i \in X_i = \{x_{i1}, x_{i2}, \ldots, x_{is}\}$ to reduce the probability of traffic accidents in sector g_i, whereas s is defined as several such measures. Every measure $x_i \in X_i$ is defined by economic, technical, social, and other values. Values $p_{ik}(x_i)$ can be calculated based on a model, for example, a non-linear model introduced in (Saaty *et al.*, 1985).

The problem itself requires creating a plan of measures for reducing the number of traffic accidents in every g_i-sector under conditions of limited resources. These measures revolve around not only reducing the amount of traffic accidents but also reducing the amount of human casualties, financial and material losses as a result of such accidents. Formally, we can present this problem of planning how to deal with critical areas and minimizing the risk of traffic accidents in the following way:

$$R(x) = \sum_{i\in I}\sum_{k\in K} a_k p_{ik}\left(x_i\right) \to \min\,, \tag{11}$$

under conditions:

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$$b_m(x) = \sum_{i \in I} b_{im}(x_i) \le b_m^*,\ m \in M, \tag{12}$$

$$x = \left(x_{1_s}, \ldots, x_{i_s}, \ldots, x_{n_s}\right) \in X = \prod_{i \in I} X_i,\ i \in I, \tag{13}$$

where $m \in M = \{1, 2, \ldots, m^*\}$ is defined as an index of expenditure for different resources required to implement measures to reduce the accident rate, b_m^*, $m \in M$ is defined as the given value of constraints of m-resource, a_k is defined as the value of losses as a result of a traffic accident involving transport category $k \in K$.

To solve problems (11) – (13), we can use algorithms described in (Shvartsman, 2023), since these problems are additive problems of discrete programming with multiple restrictions.

Solution algorithm. Definition of losses a_k, $k \in K$ caused by traffic accidents is a completely separate problem, which requires calculation of value of human lives lost during such accidents, as well as losses related to treatment of relevant traumas and repair of damaged vehicles, road objects, etc.

In order to effectively develop measures for reducing the number of traffic accidents, we must considerthe human factor, which greatly affects the rate of accidents and deaths. All traffic fatalities have different causes, which can be identified by analyzing statistical data of the Ukrainian Ministry of Internal Affairs (Vedmedenko, 2023) (Table 1). Among the 28 possible causes described in (Vedmedenko, 2023), the major ones are speeding and non-compliance with regulations related to maneuvering on the road with a total of 60% of all cases.

Table 1. Traffic accidents involving human casualties divided by causes, number, and rate in 2022

Cause	Number	Rate, %
Speeding	7561	40,25
Non-compliance with regulations related to maneuvering on the road	3846	20,47
Non-compliance with regulations related to driving through crossings	1467	7,81
Not keeping a safe distance	843	4,49
Crossing the contraflow lane	418	2,23
Drunk driving	790	4,21
Crossing the road in undesignated areas (applied to pedestrians)	593	3,16
Speeding on turn	469	2,49
Non-compliance with regulations related to driving through cross-walks	1443	7,68
Non-compliance with traffic signals of a traffic policeman	236	1,25
Other (rate below 1% for each)	1118	5,96

We define the problem of reducing the number of traffic accidents resulting in fatalities or traumas as a problem with high priority and analyze the mathematical presentation of this problem.

We define equation $D = \{1, 2, \ldots, d^*\}$ as a multitude of reasons for traffic accidents with casualties occurring on the road in question. Equation D_i, $i \in I$, $D_i \subseteq D$ on the other hand is defined as a multitude of reasons for traffic accidents for the g_i-sector of the road. For every reason $d \in D_i$ in sector g_i, $i \in I$ we calculate coefficient ρ_{di}, which is defined as the frequency

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of traffic accidents with casualties occurring in this sector. We perform these calculations based on statistical data, for example, data from Table 1.

To evaluate the effectiveness of measures x_i, $i \in I$ to reduce the rate of traffic accidents in each sector g_i, $i \in I$ we use the utility function $f_{di}(x_i) \in [0; 1]$, which can be described as a function to reduce the rate of traffic accidents for every cause $d \in D_i$. Implementing this function yields the following results depending on conditions:

– $f_{di}(x_i) = 0$, if measures x_i were not able to reduce the rate of traffic accidents with casualties due to the cause $d \in D_i$;

– $f_{di}(x_i) \in (0; 1)$, if measures x_i were able to reduce the rate of traffic accidents with casualties due to the cause $d \in D_i$;

– $f_{di}(x_i) = 1$, if no traffic accidents with casualties due to the cause $d \in D_i$ were registered in sector g_i.

In this scenario, we can formulate the problem regarding minimization of fatalities and casualties in the following way:

$$R(x) = \sum_{d \in D} \sum_{i \in I} \sum_{k \in K} \rho_{di} h_k N_{ik} \left(1 - f_{di}(x_i)\right) \to min, \tag{14}$$

under conditions that:

$$b_m(x) = \sum_{i \in I} b_{im}(x_i) \le b_m^*,\ m \text{ X } M, \tag{15}$$

$$x = \left(x_{1_s}, \ldots, x_{i_s}, \ldots, x_{n_s}\right) \in X = \prod_{i \in I} X_i,\ i \text{ X } I \tag{16}$$

where N_{ik} is defined as the average number of traffic accidents with casualties in sector $i \in I$ for vehicle category $k \in K$; h_k is defined as the average number of individuals in vehicles of category $k \in K$.

Problem (14) – (16) is a modified version of problem (11) – (13). We can define this problem as a problem of discrete programming with multiple limitations. To solve this problem we can use sequential analysis of variants or digital methods (Shvartsman, 2023). Since every transport network is a crucial part of critical infrastructure, we deem modeling processes capable of increasing traffic safety and minimizing traffic accidents an important task.

Optimization of methods of nondestructive testing for identification of damage to roads and other components of road infrastructure

Any defects in public roads and other elements of road infrastructure, such as bridges, tunnels, and overhead roads, adversely affect normal operating conditions. Among such defects are wear and tear of road coating and water isolation, cracks and a large number of holes, plastic deformation, trembling of flexible elements, damaged protective layer and cracks in reinforced concrete structures and fences, damaged expansion dams, destabilization of construction elements, etc. (Shaikh *et al.*, 2022).

The aforementioned defects often appear as a result of natural occurrences capable of affecting the condition of the road. Alternatively, they can be caused by violation of normal operating conditions, for example overloading the road or lack of proper maintenance.

Methods of nondestructive testing effectively identify these defects. Apart from advantages, there are certain disadvantages, since the results of implementing such methods do not always reflect the real state of the object due to the inability to identify all kinds of present defects (Wang *et al.*, 2022).

For instance, using methods of visual control to analyze the condition of the road coating allows to quickly identify visible surface damage, but prevents from identifying any

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damage or deformation of deeper layers. As such, analysis of any given road sector may not be objective, thus the efforts directed to cope with the identified defects become less effective, outright ineffective, or even worsen the situation.

This issue can be addressed by using different types of methods of nondestructive testing. This is common practice during research of complex systems which allows us to greatly speed up and increase the probability of defect identification. In addition, it helps to reduce the risk of partial identification (Sidliarenko, 2014). Thus, it is possible to obtain exhaustive information regarding the present damage, as well as compliance with regulations of road safety. The methods of nondestructive testing also allow us to analyze the state of road infrastructure without decommissioning any of its elements. Finally, accumulated statistical data can be used during maintenance and repair, while also helping to avoid possible mistakes during future follow-up inspections (Prokopev *et al.*, 2019).

Formulation and analysis of the problem. Assume that $L = \{l_i,\ i \in I = \{1, 2, \ldots, i^*\}\}$ is a finite set of defect types in road sectors and other types of road infrastructure, whereas I is defined as a set of damage type indexes. Defects $l_i \in L$ are described according to different criteria, such as geometry (including length and depth of a crack, evenness of road coating, etc.), complexity, probability of causing a traffic accident, etc.

To evaluate the defects $l_i \in L$, we use different methods of nondestructive testing. These methods differ by probability of identifying the defect, cost, or the necessity to utilize additional resources. For example, the effectiveness of visual control depends on qualification of experts responsible for evaluation of damage and unevenness of upper road layers, quality of asphalt, and other attributes. The aforementioned method is both the fastest and the cheapest compared to many others, which require not only specialized tools but also long-term laboratory analysis.

An alternative method revolves around utilizing space monitoring to analyze the state of road coating and road infrastructure, yet it requires additional equipment and trained personnel.

We can present a set of j-methods of nondestructive testing as the equation $M_j = \{m_{jk}\}$, where $j \in J$ is a set of indexes of methods of nondestructive testing, and $k \in K_j \subseteq K$ is a set of indexes of implementation mediums related to methods of nondestructive testing.

Equation $p(m_{jk})$ will serve as an a priori assessment of probability of damage identification by method m_{jk}. The value of the probability of damage identification $p(m_{jk})$ may be specified beforehand in technological documents, defined by experiment, or simply taken from the same research. R_i is defined as a set of alternative means of damage identification $l_i \in L$. Equation $i \in I$ can then be transformed in the following way:

$$R_i = \{r_i^1, r_i^2, \ldots, r_i^{\tau i}\},$$

where elements of set R_i are fixed technologies of damage identification r_i. On their own, they are a combination of indexes $j \in J$ of sets of different types of methods of nondestructive testing. They are written as $r_i = \{j_1, \ldots, j_s\}$, $j_1, \ldots, j_s \in J_i \subseteq J$. In this scenario, R_i can be described in the following way:

$$R_i = \begin{pmatrix} j_1^1 & j_2^1 & \cdots & j_{s_1}^1 \\ j_1^2 & j_2^2 & \cdots & j_{s_2}^2 \\ \cdots & \cdots & \cdots & \cdots \\ j_1^{\tau_i} & j_2^{\tau_i} & \cdots & j_{s_{\tau_i}}^{\tau_i} \end{pmatrix},\ j_i^1, j_{si}^1, \ldots, j_1^{\tau i}, \ldots, j_{s\tau i}^{\tau i} \in J_i\,. \qquad (17)$$

Thus, a combination $r_i = \{j_1, \ldots, j_s\} \in R_i$ is defined as a fixed technology of damage identification $l_i \in L$, which utilizes certain nondestructive testing methods. This technology is

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the most effective when used after factoring in specifics of the analyzed road sector, as well as technical parameters and operating specifications of tools used for nondestructive testing, including whether or not they can be used in conjunction with each other.

Any technology $r_i \in R_i$ can be represented as possible combinations of types (complex) of methods of nondestructive testing $v_{ri} = (m^i_{j1k}, \dots, m^i_{jsk}) \equiv (m_{jk})_{j\text{X}ri}$. They are used to identify damage $l_i \in L$.

Assuming that $V_{r_i} = \prod_{j \in r_i} M_j$ is a set of possible options of implementing technology $r_i \in R_i$, $V_i = \bigcup_{r_i \in R_i} V_{r_i}$ will be defined as a set of all alternative methods of nondestructive testing used to identify partial damage $l_i \in L$, while $V = \prod_{i \in I} V_i$ is a set of alternative methods of nondestructive testing used to identify all possible damage damage $l_i \in L$, whereas $v = (v_{r1}, \dots, v_{ri}, \dots, v_{ri*}) \equiv (v_{ri})_{i\text{X}I}$, $v_{ri} \in V_i$, $i \in I$, $v \in V$.

We define $P_i\,(p(v_{ri}))$ as probability of avoiding an emergency situation assuming that damage $l_i \in L$ was identified and dealt with (Donchenko, 2022). In this scenario, $P_i\,(p(v_{ri})) = 1 - p(v_{ri})$, where $p\left(v_{r_i}\right) = 1 - \prod_{j \in J_i}\left(1 - p\left(m_{jk}\right)\right)$ is defined as probability of identifying damage $l_i \in L$ while using complex v_{ri} for technology r_i.

Here, we analyze the problem related to identifying all defects at once from the available set L. First, we assume that $l_i \in L$ are already known, and different complexes $v_{ri} \in V_i$ are used for their identification. In this scenario, the probability of avoiding an emergency situation can be presented in the following way:

$$P_L(v) = \prod_{i \in I} p\left(v_{r_i}\right). \tag{18}$$

Value $P_L(v)$ is a lower estimate of probability to identify a set of defects L. This value can be used as a criterion to choose methods of nondestructive testing used to identify all defects $l_i \in L$ at once.

Assume that C_0 budget is provided to implement methods of nondestructive testing (including purchase and/or usage of tools, development of methods, personnel training, etc.). Thus, a problem related to developing the complex of methods of nondestructive testing can be presented in the following way:

$$P_L(v) = \prod_{i \in I} p\left(v_{r_i}\right) \to max \tag{19}$$

$$C\left(v\right) = \sum_{i \in I} \sum_{j \in J} c_{ji}\left(v_{r_i}\right) \le C_0, \tag{20}$$

$$v \equiv (v_{ri})_{i\text{X}I},\ v_{ri} = (m^i_{j1k}, \dots, m^i_{jsk}) \in V_i,\ k \in K_j \subseteq K,\ v \in V = \prod_{i \in I} V_i, \tag{21}$$

where c_{ji} is defined as the volume of funding required to implement option v_{ri}.

We can define problems (19) – (21) as problems of discrete programming with multiple limitations. To solve this problem we can use algorithms of sequential analysis of variants or digital methods (Zhang *et al*., [2020). Solving this problem results in development of complex of different types of methods of nondestructive testing $v^* = (v^*_{r1}, \dots, v^*_{ri}, \dots, v^*_{ri*})$, where $v^*_{r1} = (m_{j1i}, \dots, m_{jsi})$, $C(v^*)$ is defined as resources and $Pl(v^*)_L(v^*)$ is defined as probability.

We can improve the aforementioned mathematical model by conducting additional evaluation of severity of defects or the state of structure targeted by methods of

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nondestructive testing. In addition, we can simplify the choice of methods of nondestructive testing for cases, which do not require complex solutions.

Results and discussion

To prove the effectiveness of analyzed mathematical models used for planning road construction, reconstruction, and repair, including all relevant infrastructure, under conditions of uncertainty and reduced funding, we have conducted multiple calculations. Acquired results allow us to conclude that improving road safety and reducing the rate of traffic accidents require implementation of special measures, aimed at reducing the number of such accidents by 40-60%. The price of these measures should remain within the established limits.

We have analyzed a bridge structure as an example of planning the measures aimed at dealing with critical areas on roads, in combination with further optimization of methods of nondestructive testing used to identify damage in road coating and components of road infrastructure. As primary targets of methods of nondestructive testing, we can name asphalt-concrete roads, reinforced concrete structures, welds, and other parts. To identify set of defects L in bridge structure, we can use radio-wave methods of nondestructive testing ($M_1 = \{m_{1k}\}, k \in K_1$), acoustic methods of nondestructive testing ($M_2 = \{m_{2k}\}, k \in K_2$), radioscopy methods ($M_3 = \{m_{3k}\}, k \in K_3$), methods of visual control ($M_4 = \{m_{4k}\}, k \in K_4$). The probability of identifying defects by applying the aforementioned methods in the classic sense was the following: $P_{L1} = 0{,}913$; $P_{L2} = 0{,}908$; $P_{L3} = 0{,}963$; $P_{L4} = 0{,}891$.

Optimization of these methods results in the alteration of every complex. These complexes v^*_{ri} will transform into $v^*_{r1} = (m_{1k1}, m_{2k2}, m_{3k3}, m_{4k4})$, where $k_1 \in K_1 \subseteq K$, $k_2 \in K_2 \subseteq K$, $k_3 \in K_3 \subseteq K$, $k_4 \in K_4 \subseteq K$ are defined as chosen k-modifications of relevant methods of nondestructive testing used to identify i-defect. In this case, the low value of P_H-probability rate used to identify such defects equals 0,967.

By comparing our results with those described in (Somy *et al.*, 2022), we have deemed them congruent, and thus we can conclude that our models are not only correct but also applicable for quick calculations. Among other similar models, they can also be integrated into the programs used for planning and organization of road work.

Conclusions

In this research, we have highlighted that reduction of funding for road construction, reconstruction and repair is one of the main reasons for alteration or reallocation of funds for the ongoing projects, as well as for the projects yet to be started. Thus, mathematical modelling of such road construction projects allows to develop a plan of acquiring enough funds under conditions of reduced funding and allows to alter volumes of such funding by analyzing the object state, whereas an expert analysis of importance or project completion status are taken into account.

Reducing the amount of accident clusters can be achieved by minimizing the amount of traffic accidents in general, as well as minimizing the amount of deaths as a result of such traffic accidents. To solve mathematical problems related to reduction of accident clusters and minimizing the risk of traffic accidents, as well as to model the scenarios of increased safety, it is necessary to use a plan for implementing the measures to reduce the number of such traffic accidents by 40-60%. The aggregate cost of implementing the aforementioned means must not go beyond the defined limits.

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The analyzed problem of nondestructive testing is based on the optimal choice of methods of nondestructive testing whenever it is needed to identify an entire set of defects L. Probability detection rate P_L, which has increased from 0,891 до 0,967, allows evaluation of the process of identification of defects.

The upcoming research conducted within the scientific field analyzed in this paper will focus on the results of an implementation of analyzed mathematical models as a mathematical supplement for analytical and information systems for monitoring the state of roads. These systems will focus on supporting decision-making regarding the development of construction and maintenance plans, road repair and maintenance of current public roads, and improvement of operational facilities and structures. The aforementioned systems are important to support general road infrastructure according to established development strategy, as well as to improve road networks and implement state, social or purpose-oriented programs related to the road industry.

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МАТЕМАТИЧНІ МОДЕЛІ БУДІВНИЦТВА, РЕКОНСТРУКЦІЇ ТА РЕМОНТУ ДОРІГ ЗА УМОВ НЕВИЗНАЧЕНОСТІ

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Анотація. У статті наголошується на важливості математичного моделювання в сучасних умовах будівництва, реконструкції та ремонту автомобільних доріг в Україні. Основні аспекти, які роблять це актуальним, включають фінансову нестабільність, невизначеність фінансування, а також необхідність дотримання високих стандартів безпеки на дорогах під час проведення робіт. Зазначається, що змінні фактори, такі як кліматичні аномалії, динаміка транспортного руху, технологічний прогрес у будівельній галузі та соціальні виклики, пов'язані зі збільшеним навантаженням на дороги через військовий конфлікт, вимагають нових підходів та інструментів для оптимізації будівельних процесів.

Головною метою дослідження є застосування математичного моделювання для планування та виконання робіт на автомобільних дорогах і спорудах, зокрема в умовах обмеженого фінансування. Це допоможе покращити процес усунення аварійно-небезпечних ділянок та вибрати оптимальні методи неруйнівного контролю для виявлення пошкоджень у дорожньому покритті та інфраструктурі. Для досягнення цієї мети використовуються методи статистичного та системного аналізу, багатокритеріальної оптимізації та теорії прийняття рішень. Основний акцент робиться на розробленні стратегічних управлінських рішень та їх ефективності.

Показано, що використання математичного моделювання в процесі планування автомобільних дорожніх робіт дозволяє ефективно розподілити фінансові ресурси, навіть при їх обмеженому обсязі. Ця практика також дає змогу динамічно змінювати рівень фінансування, враховуючи при цьому експертні оцінки важливості та ступеня завершеності робіт на об'єкті.

В результаті проведеного планування робіт із виправлення аварійно-небезпечних ділянок доріг з акцентом на мінімізацію ризику дорожньо-транспортних пригод та застосування методів підвищеної безпеки був розроблений план впровадження конкретних заходів та засобів. Використання цих заходів передбачає зниження кількості дорожньо-транспортних пригод на аварійно-небезпечних ділянках в 1,4–1,6 разу, і це досягається без перевищення визначених фінансових обмежень.

Встановлена оцінка якості виявлення пошкоджень на дорожньому покритті, використовуючи показник імовірності виявлення різноманітних дефектів. Детально показано, які оптимізаційні заходи дозволили підвищити цей показник з 0,891 до 0,967.

У подальших наукових дослідженнях автора передбачається ретельно дослідити результати впровадження обговорюваних математичних моделей, особливо як математичної підтримки аналітично-інформаційної системи для моніторингу стану дорожньої інфраструктури.

Ключові слова: математична модель, автомобільний транспорт, автошлях, аварійно-небезпечна ділянка, метод неруйнівного контролю, умова невизначеності.

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