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MAGNETIC METHODS

Solution of the Inverse Problem of Creating a Uniform Magnetic Field in Coercimeters with Partially Closed Magnetic Systems

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Abstract—The use of a hybrid optimization technology by a swarm of particles with the evolutionary formation of swarm composition is considered together with the method of spatial integral equations in the synthesis of axially symmetric magnetic systems of coercimeters that contain ferromagnetic elements. The required uniformity of the magnetic field in the working volume is ensured by the optimal selection of the shape of pole pieces of a magnetizing device. A number of measures are proposed that allow acceleration of the synthesis process owing to shortening of the time spent for solving the problem of analyzing the distribution of the magnetic field in the working volume of the coercimeter.

Keywords: spatial integral equations, global optimum, optimization by a particle swarm, genetic algorithm, hybrid, magnetic-field source, synthesis, on/off technology.

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INTRODUCTION

The magnetic-field uniformity (MFU) in the working volume, in which a reference specimen and sensors are placed during measurements, is one of the factors that influence the measurement error in coercimetry [1–5]. The required field-uniformity level can be provided via selection of the design parameters of the magnetic system (MS) of a coercimeter (CR). However, the complex character of the dependence of the magnetic-field distribution on the values of the MS parameters excludes the possibility of selecting them by the trial-and-error method, because such an approach requires analysis of a large number of variants. This determines the necessity of using an optimal synthesis procedure with a reasonable variant-generation strategy in the solution of the considered problem. This strategy implies the wide application of computer equipment for selecting the best variant. The analysis and evaluation of the MFU in the working volume of the CR can be performed by its numerical simulation.

The problem of creating a uniform MF in the case of a CR with an open MS, in which systems of coils with currents usually serve as field sources, can be solves by determining the currents in the coils at their fixed geometrical parameters as a result of a linear synthesis [6], determining the dimensions and positions of coils via synthesis in a nonlinear formulation [7], and the use of a structural synthesis [8], which allow minimization of the number of coils used in the CR and selection of an optimal sequence for turning these coils on.

For a CR with a partially closed MS, the synthesis problem is complicated by the fact that the configuration of the magnetic field in the working volume is determined not only by the topography of the magnetizing field of coils with currents, which operate as primary sources, but also by the shapes of ferromagnetic elements (FEs) of the MS—the magnetic circuit and pole pieces that are included in a CR of a given type, and by the magnetic characteristics of the materials from which the FEs are manufactured. In this case, selecting the structural parameters of the MS with the FEs implies calculation of the magnetization distribution in them for each of the considered sets of parameter values, thus requiring considerable computing power. Therefore, within this approach to the synthesis, an algorithm for solving the analysis problem, which is harmonically matched to it, must be implemented.

In the first approximation, the magnetization distribution in the pole pieces can be considered constant, thus allowing one to significantly simplify the problem of calculating the configuration of the magnetic field, which is produced by the pole pieces in the working volume [9–13], and selecting an optimal shape of the pole pieces.

The selection of an optimization algorithm that is used to solve the synthesis problem is also a substantial factor, because the goal functions that arise in this class of problems are generally of a multiextremum

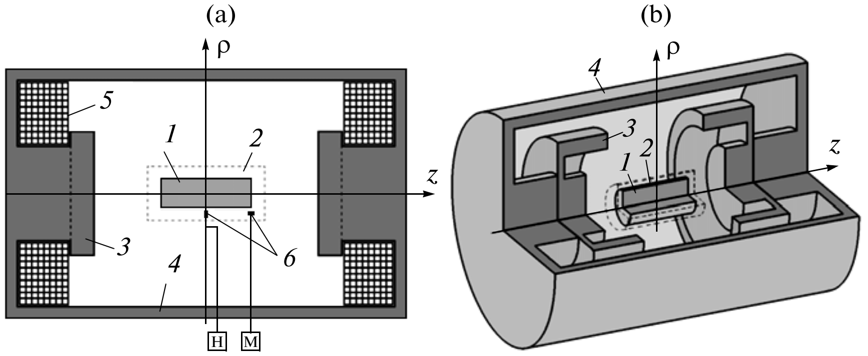

Fig. 1. External view of a coercimeter with pole pieces: (*1*) tested object, (*2*) working volume, (*3*) pole pieces, (*4*) magnetic circuit, (*5*) coils, and (*6*) Hall transducers.

and ravine character. In [14], the synthesis problem was formulated as a problem of nonlinear programming, and a large-step method for extremum seeking that allows finding of only a local solution was used. In this case, difficulties in selecting the initial approximation arise, because it substantially determines the found extremum. The method of equivalent turns as applied to the solution of this problem was developed in [15]. However, the application of this approach is described only for an idealized case of a field dependence of the magnetization of a ferromagnetic material, although, for the majority of actual MSs, the magnetic characteristics are nonlinear and there is a substantial dependence of the magnetization distribution on the magnetizing-field topography. In this study, the optimization problem was solved using classical steepest-descent methods, which also do not generally ensure finding a global solution. The problem of synthesizing MSs of isochronous cyclotrons was considered in [16]. The direct problem was solved using the finite-element method (FEM), which, despite its several advantages, has a number of shortcomings. The main shortcoming is associated with the fact that when the FEM is used it is necessary to discretize a ferromagnetic body and the space surrounding it; this leads to an increase in the dimensionality of the solved problem and implies introduction of artificial limitations of the calculated region. In addition, for new variants of the pole elements (PEs) of the electromagnet that are obtained during optimization, repeated partitioning or a deformation of the existing grid of elements is required. Particular information on the conditional-optimization method that is used was absent in [16].

At present, the sensitivity-analysis method has become widespread in designing magnetic systems. The optimal shape of the surfaces of pole pieces described with Besier and *B*-splines is sought in [17]. In this study, a combination of the sensitivity-analysis and steepest-descent methods was used to solve the synthesis problem. Although this approach allows one to choose promising trends in changing the parameters of the shape of the pole pieces, the use of the local optimization method does not guarantee the finding of their best geometry.

This study is aimed at the development of a method of optimal parametric synthesis of axially symmetric MS of coercimeters on the basis of the algorithm of global multiagent optimization using the apparatus of spatial integral equations, which allows accounting for nonlinear properties of ferromagnetic materials.

FORMULATION OF THE SYNTHESIS PROBLEM

The subject of investigation is a CR with an axially symmetric closed MS, which is used to test the magnetic properties of materials and is described in detail in [1–5]. The CR includes coils *5* (Fig. 1) and coils for pulsed magnetization (not shown in the figure), whose main function is to bring a specimen to the saturation magnetization. These coils are unnecessary for soft magnetic materials, while for hard magnetic materials they are used only at the magnetization stage. Therefore, subsequently we consider an MS, whose appearance with a specimen installed in it is shown in Fig. 1.

The solution of the synthesis problem reduces to a selection of the geometrical parameters of pole pieces that are manufactured from a ferromagnetic material with nonlinear magnetic characteristics, which provide a specified field distribution in the working volume Ω; thus, the inverse problem of magnetostatics is solved. In this case, each of the pole pieces is represented in the form of a set of cylindrical PEs, whose radii and heights serve as the sought parameters. The number of pole elements is specified a priori.

Figure 2 illustrates the varied PE parameters and presents the geometrical dimensions, which remain constant during the synthesis process for one of particular examples.

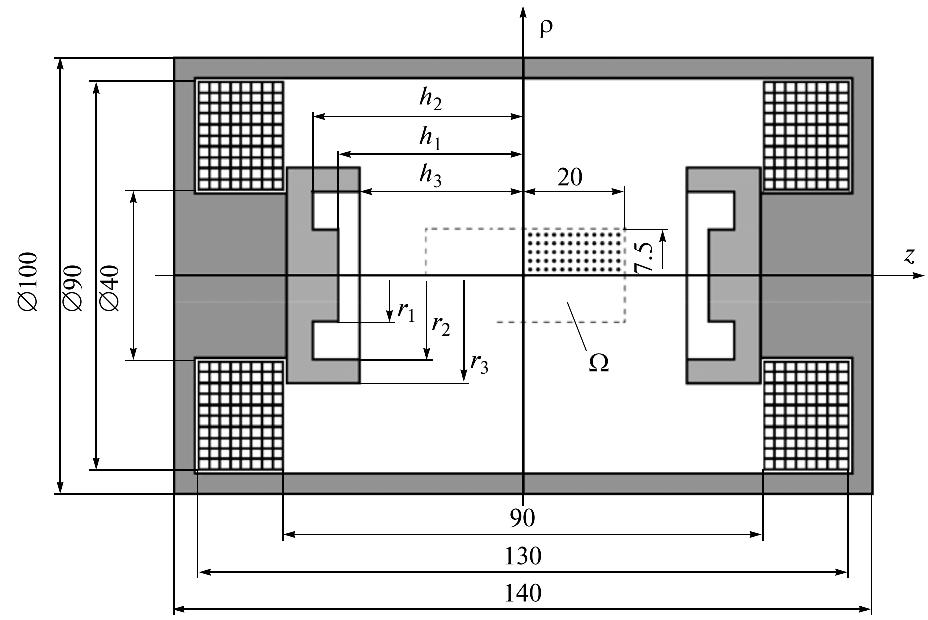

Fig. 2. Dimensions and parameters of the synthesized coercimeter with three pole elements.

During the synthesis, the set of sought parameters is represented as a vector whose components are the values of the heights h_i and radii $\rho_i N$ of the PEs:

$$\mathbf{X} = (h_1, h_2, \ldots, h_N, \rho_1, \rho_2, \ldots, \rho_N). \tag{1}$$

The CR parameters are imposed by a number of design limitations, which are specified in the form of a system of inequalities:

$$\begin{cases} h_{\min} \le h_i \le h_{\max}; \\ 0 \le \rho_1 \le \rho_2 \le \ldots \le R_{\max}, \end{cases} \tag{2}$$

where $h_{\min}$ and $h_{\max}$ are the lowest and highest acceptable values of the PE heights, and $R_{\max}$ is the maximum possible value of the pole-piece radius.

To estimate the MFU, a set of K test points is regularly positioned in the working volume. When the field is analyzed, the maximum relative deviation of the values of the magnetic-field strengths at these points from the value $\mathbf{H}_0$ at the center of the tested zone is calculated:

$$\Delta = \max_k \frac{|\mathbf{H}_k - \mathbf{H}_0|}{|\mathbf{H}_0|} 100\%, \tag{3}$$

where $\mathbf{H}_k$ is the field strength at the kth test point, $k = \overline{1, K}$.

Because the coercimeter's MS is axially symmetric, it is sufficient to analyze the field distribution only in a quarter of the working-volume cross section (Fig. 2). Note that the working volume Ω must exceed the volume occupied by the specimen that is studied in the CR. This is determined by the necessity of placing sensors in the region of a uniform field and allows a decrease in the error associated with an inaccuracy in positioning the specimen in the device.

As the goal function, let us consider the functional

$$f(\mathbf{X}) = \sum_{k=1}^{K} (H_z - H_0)^2 + \sum_{k=1}^{K} (H_\rho)^2, \tag{4}$$

whose minimization ensures an increase in the degree of the MFU in the working volume. To solve the global-optimization problem, it is promising to use comparatively new bionic evolutionary optimization technologies, such as genetic algorithms (GAs) [18–20] and the multiagent optimization by a particle swarm (PSO) [21–23], which is based on the swarm intelligence paradigm. Each of these methods has its advantages and flaws; therefore, this paper proposes a developed hybrid optimization technology by a particle swarm with evolutionary formation of the swarm composition as the optimization method. The combined use of these methods makes it possible to increase both the possibility of finding the global solution and the algorithm convergence rate.

One characteristic feature of the hybrid algorithm is a much smaller number of calculations of the goal function than that when each algorithm is used independently. This is especially important when performing synthesis of an MS, which requires multiple solutions of a resource-intensive analysis problem. Detailed information on the hybrid algorithm, verification, and analysis of its convergence for multidimensional ravine and multiextremum goal functions are presented in [24], where all advantages of the developed global-optimization method are demonstrated. Examples of the efficient use of the algorithm are also presented in [25], where it was used to synthesize axially symmetric MSs of electromagnets with a specified field distribution in the working zone in the case where the linear character of the dependence between the magnetic-field strength and the ferromagnet magnetization was taken into account. Under such assumptions, the field-synthesis problem was solved using the method of boundary integral equations.

SOLUTION OF THE PROBLEM OF ANALYZING THE FIELD IN THE MAGNETIC SYSTEM OF THE COERCIMETER

The nonlinear dependence of the magnetization on the field strength can be taken into account as a result of the application of the apparatus of spatial integral equations (SIE), as was done in [26–29], which are adapted to the case of axially symmetric MSs. The magnetic field created in the working volume of the CR consists of two components: a field $\mathbf{B}'$ from magnetized ferromagnetic parts of the MS and a field $\mathbf{B}_0$ from external sources:

$$\mathbf{B} = \mathbf{B}' + \mathbf{B}_0. \tag{5}$$

The field that is created by the coils with allowance for their rectangular cross section can be calculated from the known numerical and analytical formulas [30]. In order to calculate the magnetic field created by the FEs of the CR, the magnetization values in them must be determined. The distribution of the magnetization $\mathbf{M}$ in the FEs of the CR can be determined as a result of the solution of the SIE

$$\mathbf{B}(Q) = \frac{\mu_0}{4\pi}\mathrm{rot}_Q\iiint_V \mathbf{M}(P) \times \mathrm{grad}\frac{1}{\mathbf{r}_{PQ}}dV_P + \mathbf{B}_0(Q), \tag{6}$$

where $\mathbf{r}_{PQ}$ is the vector drawn from the source point P to the observation point Q, $\mathbf{B}(Q)$ is the magnetic induction at the point Q, and $\mathbf{B}_0(Q)$ is the magnetic induction of the external field that is produced by the currents flowing in the coils. If the magnetic-circuit body consists of n arbitrary parts, Eq. (6) is written in the form

$$\mathbf{B}(Q) = \frac{\mu_0}{4\pi}\sum_{j=1}^{n}\mathrm{rot}_Q\iiint_V \mathbf{M}(P) \times \mathrm{grad}\frac{1}{\mathbf{r}_{PQ}}dV_P + \mathbf{B}_0(Q). \tag{7}$$

Using the known relationships of vector analysis and the curl theorem and introducing the designations for the bulk and surface current densities $\mathbf{j}(P) = \mathrm{rot}_P\mathbf{M}(P)$ and $\mathbf{i}(P) = -\mathbf{n}_P \times \mathbf{M}(P)$, respectively, integral equation (7) can be transformed as follows:

$$\mathbf{B}(Q) = \frac{\mu_0}{4\pi}\sum_{j=1}^{n}\mathrm{rot}_Q\iiint_V \frac{\mathbf{j}(P)}{\mathbf{r}_{PQ}} + \frac{\mu_0}{4\pi}\sum_{j=1}^{n}\mathrm{rot}_Q\oint_{S_j}\frac{\mathbf{i}(P)}{\mathbf{r}_{PQ}}dS_P + \mathbf{B}_0(Q). \tag{8}$$

If only bodies with axial geometries that are partitioned into circular elements are considered (Fig. 3) and it is assumed that the magnetization vector within each such element is constant: $M_\rho = \mathrm{const}$, $M_z = \mathrm{const}$, and $M_\varphi = 0$, then the expression for $\mathrm{rot}_P\mathbf{M}(P)$ written in a cylindrical coordinate system clearly shows that the bulk current density $\mathbf{j} = 0$.

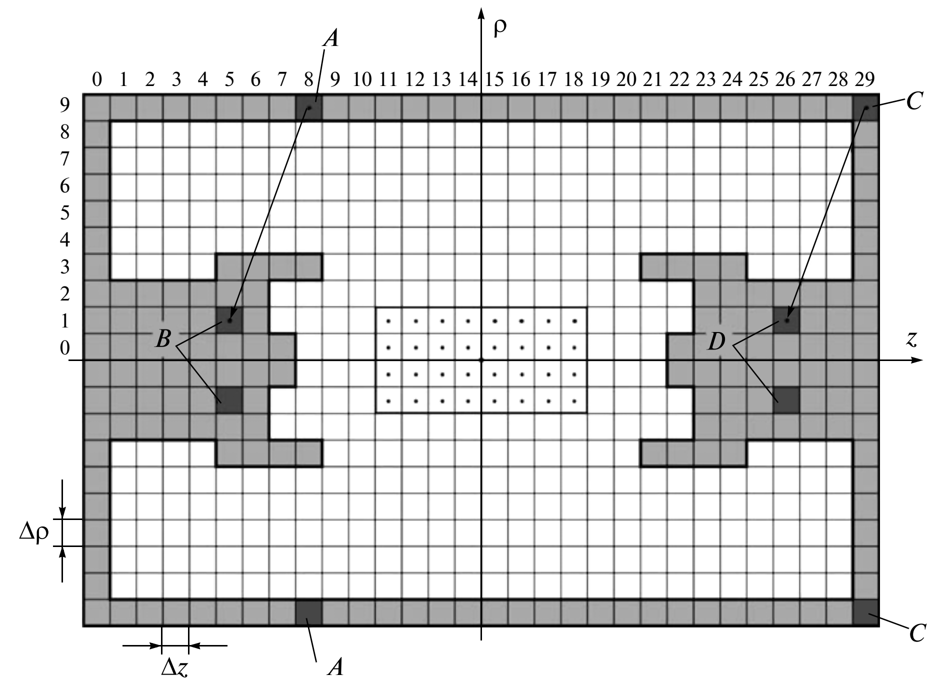

Fig. 3. Discretization of the magnetic system of the coercimeter in to ring-shaped elements: *A*, *B* and *C*, *D* are the cross sections of ring-shaped partition elements with different diameters.

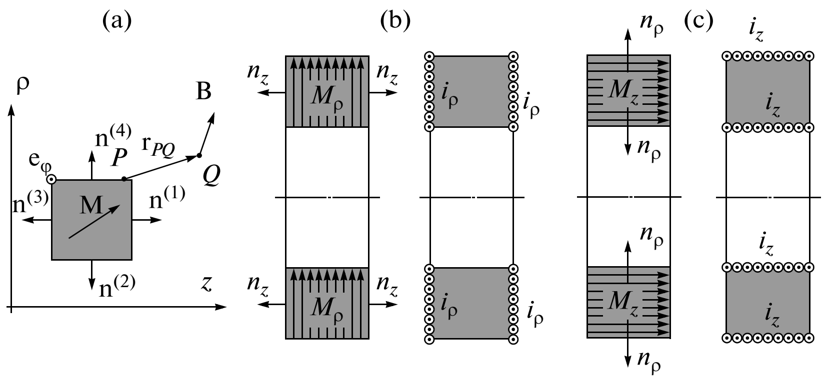

Fig. 4. Replacement of partition elements by equivalent coils: (a) graphical image of a partition element, (b) account for the radial component, and (c) account for the axial component.

Placing the observation point consecutively at the centers of the cross sections of the partition elements, let us write a discrete analogue of the integral equation

$$\mathbf{B}(Q) = \frac{\mu_0}{4\pi}\sum_{j=1}^{n} \mathrm{rot}_Q \oint_{S_j}\int \frac{\mathbf{i}(P)}{\mathbf{r}_{PQ}} dS_P + \mathbf{B}_0(Q). \tag{9}$$

After the curl is calculated at points Q_i, formula (9) takes the form

$$\mathbf{B}(Q_i) = \frac{\mu_0}{4\pi}\sum_{j=1}^{n} \mathrm{rot}_Q \oint_{S_j}\int \frac{\mathbf{i}(P)\times \mathbf{r}_{PQ_i}}{r_{PQ_i}^3} dS_P + \mathbf{B}_0(Q). \tag{10}$$

The integral in Eq. (10) is formally equal to the magnetic induction produced by thin coils with surface current densities $\mathbf{i}(P)$. Therefore, writing it in the form of the sum of the integrals for each surface of partition elements (Fig. 4), we obtain the formula

$$\mathbf{B}(Q_i) = \frac{\mu_0}{4\pi}\sum_{j=1}^{n}\sum_{k=1}^{4}\oint_{S_j^{(k)}}\int\frac{\mathbf{i}_j^{(k)}\times\mathbf{r}_{PQ_i}}{r_{PQ_i}^3}dS_P + \mathbf{B}_0(Q), \tag{11}$$

where $\mathbf{i}_j^{(k)} = -\mathbf{n}_j^{(k)}\times\mathbf{M}_j$ is the surface current flowing over the kth surface of the jth partition element.

The magnetization vector $\mathbf{M}_j$ can be expanded in terms of its components

$$\mathbf{M}_j = \mathbf{M}_{\rho j}\mathbf{e}_\rho + M_{zj}\mathbf{e}_z. \tag{12}$$

Taking into account that

$$\mathbf{n}_j^{(1)} = \mathbf{e}_z,\quad \mathbf{n}_j^{(2)} = -\mathbf{e}_\rho,\quad \mathbf{n}_j^{(3)} = -\mathbf{e}_z,\quad \mathbf{n}_j^{(4)} = -\mathbf{e}_\rho, \tag{13}$$

the following expressions are valid for the surface currents:

$$\begin{aligned}\mathbf{i}_j^{(1)} &= -M_{\rho j}\mathbf{e}_z\times\mathbf{e}_\rho = -M_{\rho j}\mathbf{e}_\varphi,\quad \mathbf{i}_j^{(2)} = M_{zj}\mathbf{e}_\rho\times\mathbf{e}_z = -M_{zj}\mathbf{e}_\varphi,\\ \mathbf{i}_j^{(3)} &= M_{\rho j}\mathbf{e}_z\times\mathbf{e}_\rho = M_{\rho j}\mathbf{e}_\varphi,\quad \mathbf{i}_j^{(4)} = -M_{zj}\mathbf{e}_\rho\times\mathbf{e}_z = M_{zj}\mathbf{e}_\varphi.\end{aligned} \tag{14}$$

Therefore, integral Eq. (11) allows representation in the form

$$\mathbf{B}(Q_i) = \mu_0\sum_{j=1}^{n}(\Theta_{ij}M_{\rho j} + \Xi_{ij}M_{zj}) + \mathbf{B}_0(Q_i), \tag{15}$$

where the following designations are introduced:

$$\begin{aligned}\Theta_{ij} &= \frac{1}{4\pi}\oint_{S_j^{(3)}}\int\frac{\mathbf{e}_\varphi\times\mathbf{r}_{PQ_i}}{r_{PQ_i}^3}dS_P - \frac{1}{4\pi}\oint_{S_j^{(1)}}\int\frac{\mathbf{e}_\varphi\times\mathbf{r}_{PQ_i}}{r_{PQ_i}^3}dS_P,\\ \Xi_{ij} &= \frac{1}{4\pi}\oint_{S_j^{(4)}}\int\frac{\mathbf{e}_\varphi\times\mathbf{r}_{PQ_i}}{r_{PQ_i}^3}dS_P - \frac{1}{4\pi}\oint_{S_j^{(2)}}\int\frac{\mathbf{e}_\varphi\times\mathbf{r}_{PQ_i}}{r_{PQ_i}^3}dS_P.\end{aligned} \tag{16}$$

The values of the terms on the right-hand sides of Eqs. (16) are numerically equal to the field strengths created at a point Q_i by the surface currents of unit density that flow over the corresponding lateral surfaces of a circular partition element and the directions of which at each surface point are specified by the vector $\mathbf{e}_\varphi$.

The components of the magnetic-field strength, which are created by a thin turn carrying a unit current with a radius ρ_P and a coordinate z_P, are determined from the formulas presented below [30]:

$$\begin{aligned}h_\rho(Q) &= \frac{1}{2\pi}\left[E(k)\frac{\rho_P^2+\rho_Q^2+(z_Q-z_P)^2}{(\rho_P-\rho_Q)^2+(z_Q-z_P)^2} - K(k)\right]\frac{z_Q-z_P}{\rho_Q\sqrt{(\rho_P+\rho_Q)^2+(z_Q-z_P)^2}},\\ h_\rho(Q) &= \frac{1}{2\pi}\left[E(k)\frac{\rho_P^2-\rho_Q^2-(z_Q-z_P)^2}{(\rho_P-\rho_Q)^2+(z_Q-z_P)^2} + K(k)\right]\frac{1}{\sqrt{(\rho_P+\rho_Q)^2+(z_Q-z_P)^2}},\\ k &= \sqrt{\frac{4\rho_Q\rho_P}{(\rho_P+\rho_Q)^2+(z_Q-z_P)^2}},\end{aligned} \tag{17}$$

where $K(k)$ and $E(k)$ are full elliptic integrals of the first and second kinds. Then the integrals in expressions (16) can be calculated by integrating the vector $\mathbf{h}(Q) = h_\rho(Q)\mathbf{e}_\rho + h_z(Q)\mathbf{e}_z$ along the corresponding segments $l_j^{(k)}$ that form the contour of the rectangular cross section of the jth partition element:

$$\frac{1}{4\pi}\oint_{S_j^{(k)}}\int\frac{\mathbf{e}_\varphi\times\mathbf{r}_{PQ_i}}{r_{PQ_i}^3}dS_P = \int_{l_j^{(k)}}\mathbf{h}(Q_i)dl_P. \tag{18}$$

Introducing the operator of the inverse magnetic characteristics $\mathbf{H} = F^{-1}(\mathbf{M})$ that allows calculation of the magnetic-field strength in the FEs from their magnetization, we obtain for the magnetic-induction vector

$$B = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(F^{-1}(\mathbf{M}) + \mathbf{M}). \tag{19}$$

Thus, integral Eq. (10) can be replaced by the system of nonlinear algebraic equations

$$F^{-1}(\mathbf{M}_i) + \mathbf{M}_i = \sum_{j=1}^{n}(\Theta_{ij}M_{\rho j} + \Xi_{ij}M_{zj}) + \mathbf{H}_{0i}, \tag{20}$$

where $\mathbf{H}_{0i}$ is the vector of the magnetic-field strength created by the coils at the center of the rectangular cross section of the ith circular partition element, $i = \overline{1, n}$.

This system of equations can be written in the matrix form

$$\mathbf{F}^{-1}(\mathbf{M}) + \mathbf{M} = \mathbf{A} \cdot \mathbf{M} + \mathbf{H}_0. \tag{21}$$

Influence matrix $\mathbf{A}$ consists of blocks with a size of 2×2 in the form

$$A_{ij} = \begin{pmatrix} \Theta_{ij\rho} & \Xi_{ij\rho} \\ \Theta_{ijz} & \Xi_{ijz} \end{pmatrix}, \tag{22}$$

such that

$$\Theta_{ij}M_{\rho j} + \Xi_{ij}M_{zj} = A_{ij} \cdot \mathbf{M}_j = \begin{pmatrix} \Theta_{ij\rho} & \Xi_{ij\rho} \\ \Theta_{ijz} & \Xi_{ijz} \end{pmatrix}\begin{pmatrix} M_{\rho j} \\ M_{zj} \end{pmatrix}. \tag{23}$$

To describe the magnetic characteristic in subsequent calculations, it is convenient to use the formula that provides the reversibility of the characteristic at a field value [29]:

$$M(H) = \lambda M_1(H) + (1 - \lambda)M_2(H), \tag{24}$$

where $M_1(H)$ is the equation of the magnetic characteristic that describes nonlinear magnetic properties of the substance, $M_2(H) = \chi H$, and λ is the introduced parameter that assumes values from the segment [0, 1]. The value of χ is chosen so as to obtain the best approximation of a segment of the nonlinear magnetic characteristic by a linear dependence. The nonlinear magnetic characteristic is specified by the analytical expression [31]

$$M_1(H) = \chi_H\frac{H_{cs}^2 H}{H^2 + H_{cs}^2} + \frac{M_s}{\pi}\frac{H^2}{H^2 + \alpha H_{cs}^2}\left(\arctan\frac{H_{cs} + H}{H_0} - \arctan\frac{H_{cs} - H}{H_0}\right). \tag{25}$$

After system of equations (21) is solved using the method of continuation with respect to the parameter λ, the distribution of magnetizations in partition elements, whose values allow calculation of the magnetic field in the working volume of the MS, can be found.

INFORMATION MODEL OF THE SYNTHESIS PROBLEM

Because the analysis problem is multiply solved during the synthesis procedure, it is especially important that the field calculation take as short a time as possible. The calculation of the coefficients of the influence matrix $\mathbf{A}$ and the solution of the obtained system of nonlinear equations, in which the number of unknowns may be as high as tens of thousands, are the most critical operations with respect to their execution time. In order to promote the optimization process, a grid of $N \times N_z$ discrete ring-shaped elements that covers the area of the potentially possible arrangement of MS elements is introduced on the ρOz plane. Subsequently, when the on/off technology is applied to the grid elements, an optimal MS geometry that ensures a specified field distribution in the working volume is chosen during synthesis (Fig. 3).

The vector of parameters $\mathbf{X}$ is then associated with a 2D bitmap of the MS, and 1 and 0 encode the presence of the corresponding ferromagnetic ring-shaped element of a discrete MS model and its absence, respectively.

Determining the elements of the influence matrix for various mutual positions of grid elements helps to avoid their repeated calculations at the synthesis stage for cases of MSs of different geometries. In a gen-

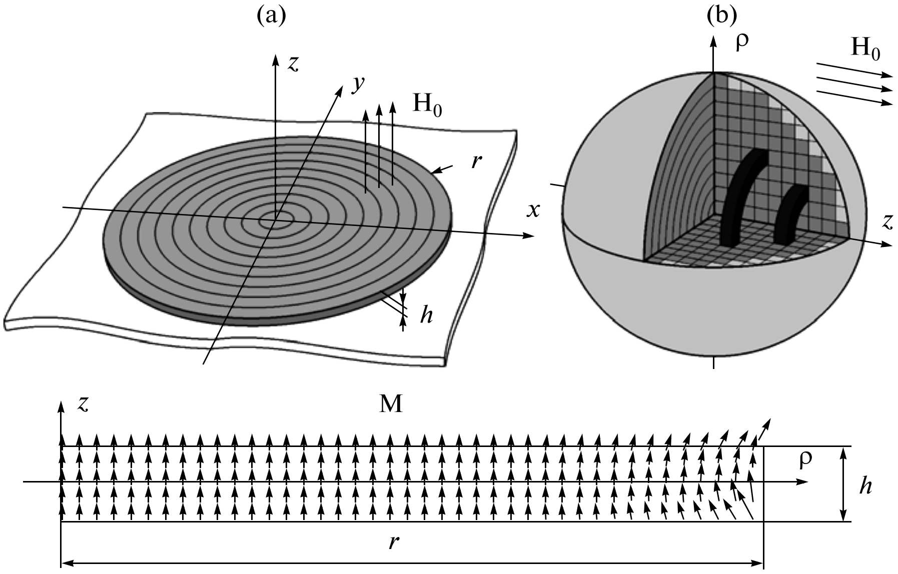

Fig. 5. Partition of (a) a thin disk and (a) a sphere into ring elements; (c) magnetization distribution in the disk.

eral case, to store the coefficients of the influence matrix **A**, $4N_\rho^2 N_z^2$ computer memory cells are required, where N_ρ and N_z are the numbers of partitions along the coordinate axes.

If the fact that the numerical values of the mutual-influence coefficients between pairs of partition elements are determined to within the sign by only three numbers (the radii of these rings and the magnitude of the distance between them) is taken into account, the pairs of rings shifted relative to one another along the axis of symmetry have equal influence coefficients. Thus, the ring-shaped partition element A has the same influence on element B as that exerted by element C on D (Fig. 3). Accounting for the translational symmetry described allows a decrease in the size of the memory that is required for storing the matrix **A** to a value of $4N_\rho^2 N_z^2$. If the points in the tested region at which the magnetic-field distribution is analyzed coincide with the centers of the elements of the introduced grid, the use of the coefficients of the influence matrix makes it possible to substantially increase the field calculation rate without executing the direct integration procedure.

To achieve high accuracy in calculating the field, it is necessary to use grids with a sufficiently small step, thus leading to a substantial increase in the order of the solved system of nonlinear equations, which is evaluated at $\approx 10^4$. An effective method for solving such systems is the Newton–GMRES method combined with the method of continuation with respect to the parameter λ, which is introduced into the problem according to formula (25). This method is based on the projection on Krylov subspaces [32, 33] and allows one to solve systems with densely filled matrices. The specific features and substantial advantages of this method are the absence of the necessity of forming an influence matrix in an explicit form, and matrix–vector products with the participation of the matrix **A** are calculated implicitly by traversing discrete elements of the grid in accordance with the bitmap content.

When solving systems of nonlinear equations, time resources can be significantly saved at the expense of using the initial approximations obtained at the previous steps of the optimization algorithm for a MS of similar geometry.

VERIFICATION OF THE INFORMATION MODEL OF THE ANALYSIS PROBLEM

The developed field-analysis algorithm with the use of SIEs was verified for the case with a known analytical solution, viz., an infinite ferromagnetic plate with linear magnetic properties, which is placed in a uniform MF with a strength $\mathbf{H}_0$ that is perpendicular to the plate surface. For this purpose, the infinite plate was replaced by a thin disk (Fig. 5) of thick $h = 5$ mm and radius $r = 100$ mm. The strength of the external field was assumed to be $H_0 = 1$ kA/m, the magnetic permeability $\mu = 100$, and the disk was partitioned with the same step of 1 mm along the ρ and z axes. The analytical dependence that allows deter-

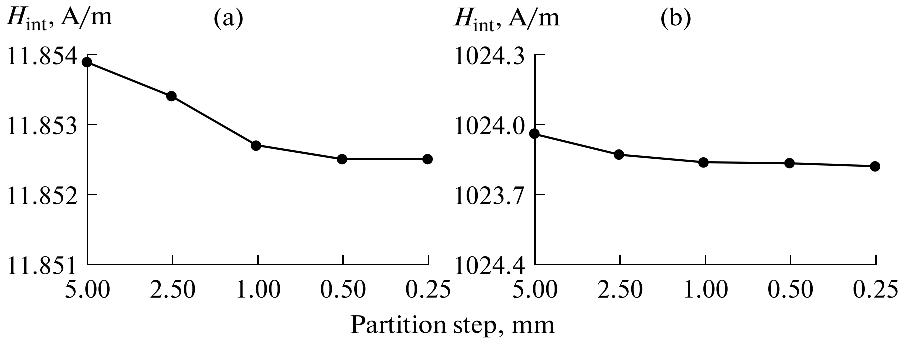

Fig. 6. Results of field calculations on the axis (a) inside and (b) outside the disk for a nonlinear case.

mination of the field strength inside the plate $H = H_0/\mu$ can be obtained from the boundary conditions for the normal component of the magnetic-induction vector.

Comparison of the results of the analytical and numerical solutions has shown that the error in determining the magnetic-field strength is within 2.1%. As the disk radius further increases to $r = 500$ mm, the calculation error is 0.45%, thus indicating the fulfillment of the condition $\lim\limits_{r \to \infty} H_{cal} = H$. In this case, the field calculated at the center of the plate has a strength $H_{cal} = 10.045$ A/m.

It is known that an infinite ferromagnetic plate is magnetized under the same conditions uniformly even at nonlinear characteristics of its material. Because a disk is an approximate model of an infinite plate, let us evaluate the MFU only in a portion that is located quite close to the disk center. Model calculations were performed for a disk geometry coinciding with the previous linear case but with allowance for nonlinear properties of the material in accordance with dependence (25) and the parameters taken from [31]. A numerical experiment revealed that, in the region restricted by the testing radius equal to one third of the disk radius, the magnetization nonuniformity is within 0.2%. As the testing radius increases, a trend toward a uniformity disturbance is observed, which is due to the influence of the disk edges.

In addition, the magnetic-field strengths inside H_{int} and outside H_{ext} the disk, which simulates the infinite plate, were calculated. H_{ext} was calculated at the point that was positioned on the z axis at a distance of 5 mm from the disk surface. Figure 6 shows the results of calculating these parameters in a nonlinear case at different disk partitioning steps. The curves show that the computational process is stable and convergent and a step discreteness of 0.5 mm is sufficient for obtaining reliable strength values of the determined field.

The analysis of the magnetization of a sphere with a radius $R = 20$ mm by a uniform field $H_0 = 1$kA/m also serves as a test of the numerical-simulation correctness; in this case, the sphere magnetization must be uniform.

It is considered that the sphere is manufactured from a material with nonlinear magnetic characteristics. Ring-shaped elements with a rectangular cross section were used as partition elements of the sphere, thus making it possible to describe its geometry only approximately. The uniformity was evaluated only within the sphere volume with a testing radius $r = 15$ mm. The partition step was 0.1 mm. The results of the numerical simulation show that the magnetization uniformity is at a level of 1.3%. Because the ring elements with different radii have different volumes and the magnetization within each of them is considered constant, it is also of interest to find the averaged magnetization uniformity of the sphere over its entire volume, which was evaluated using the formula

$$\Delta_M = \frac{\sum_{i=1}^{n} |\mathbf{M}_i - \mathbf{M}_0| V_i}{|\mathbf{M}_0| V} 100\%,$$

where $\mathbf{M}_i$ is the magnetization in the ith ring, $\mathbf{M}_0$ is the magnetization at the center of the sphere, n is the number of ring-shaped elements, V_i is the volume of the ith ring element, and V is total volume of the sphere represented by the totality of the partition elements. Calculations showed that the volume-averaged magnetization uniformity is 2.8% and the magnetic-field strength at the center of the sphere is 33.5 A/m.

Optimal parameters of synthesized magnetic systems

PE number	Model example 1		Model example 2	
	ρ, mm	*h*, mm	ρ, mm	*h*, mm
1	16	33	1	25
2	29	42	28	25
3	35	22	34	20

EXAMPLES OF THE SYNTHESIS OF MAGNETIC SYSTEMS OF COERCIMETERS

Let us consider as an example the synthesis of a coercimeter's MS using the developed method. In this case, the dimensions presented in Fig. 2 were used for the MS. The current density in the coils was chosen equal to $j = 2$ A/mm^2. The values of the PE parameters were varied within the limits $h_{\min} = 20$ mm, $h_{\max} = 45$ mm, $R_{\max} = 35$ mm. In order to evaluate the magnetic-field uniformity in the working volume Ω, 150 test points were chosen with a step of 1 mm along the ρ and z axes. The volume occupied by the MS was regularly partitioned into ring-shaped element with a step of 1 mm. The total number of partitions of the MS was ~7000.

The pair of coils used in the MS in the absence of a FE creates in the working volume a magnetic field with a strength $H_0 = 4096$ A/m and uniformities $\Delta_\rho = 18.7\%$ and $\Delta_z = 37.5\%$ for the radial and axial components, respectively. When flat cylindrical pole pieces are used in the CR, the best MFU results are attained when the poles are maximally close to the working volume, thus ensuring a field uniformity of $\Delta_\rho = 1.46\%$ and $\Delta_z = 5.31\%$ and a field $H_0 = 33?373.5$ A/m at the center of the working volume.

After performing an optimal synthesis for the first model example, the optimal parameters of the CR pole pieces that are listed in the table, at which the field strength is 21?345.4 A/m and the uniformities are $\Delta_\rho = 0.25\%$ and $\Delta_z = 0.47\%$, were found.

Since the CR operation implies a change in the current in the coils, it is of interest to investigate the magnetic-field strength and the degree of its uniformity in the working volume as functions of the current density. Figure 7 illustrates the dependences $H(j)$ and $\Delta_{\rho,z}$ for the first model example (point *B*). The analysis of the plots in Fig. 7b has shown that the MFU of the CR depends on the current density in the coils. Considering the question related to the provision of a specified degree of field uniformity with account for such a dependence is of independent interest and requires further investigation beyond the subject of this study. Presumably, taking this dependence into account during synthesis will require the formulation of the goal function as a multicriterion function.

The second example of synthesis was performed at a current density of $j = 1.5$ A/mm^2 (point *A*), with the other conditions being the same. The objective of this numerical experiment was a fundamental test of the possibility of achieving a sufficient degree of field uniformity at other current densities. The values of the optimal parameters that were found are presented in the table; they provide a field strength of

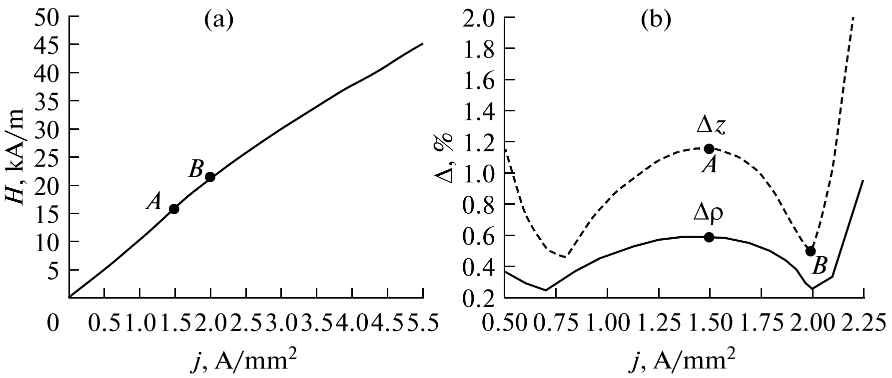

Fig. 7. Dependences of the magnetic field (a) strength and (b) uniformity in the working volume on the current density in the coils for model example 1.

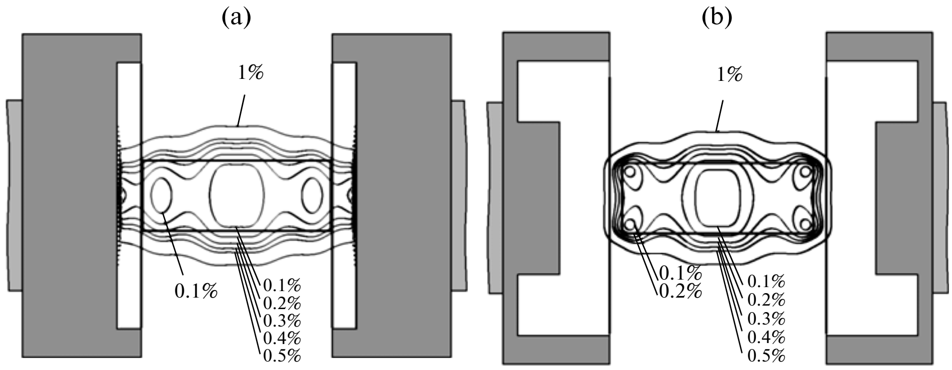

Fig. 8. Isolines of the field uniformity in the coercimeter working volume for model examples (a) 1 and (b) 2.

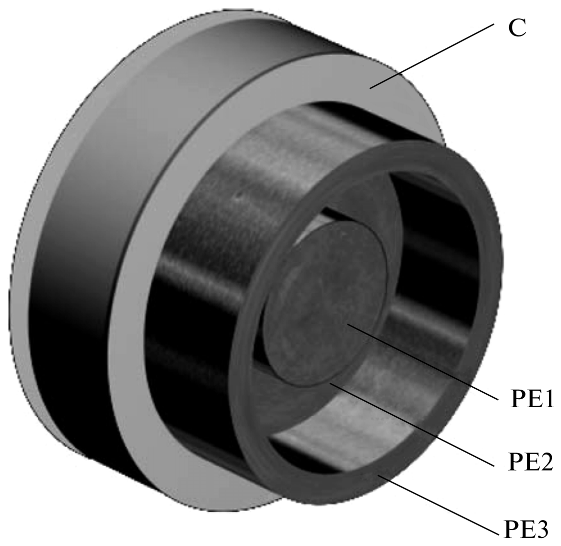

Fig. 9. 3D CAD model of a pole piece of the coercimeter's magnetic system: (C) coil and (PE) pole elements.

20600.9 A/m in the working volume and uniformities $\Delta_\rho = 0.31\%$ and $\Delta_z = 0.37\%$ for the radial and axial components of the magnetic-field strength.

Figure 8 shows the uniformity isolines for the synthesized forms of pole pieces and Fig. 9 shows a 3D CAD model of one of the synthesized poles that corresponds to the model example 2.

CONCLUSIONS

(1) The proposed synthesis method, in which the SIE apparatus, the global algorithm of the optimization by a particle swarm with evolutionary formation of the swarm composition, the on/off technology, the Newton–GMRES method for solving systems of high-order nonlinear equations, the method of continuation with respect to a parameter, and the accounting for the translational symmetry are simultaneously harmonically combined, allows efficient synthesis of axially symmetric magnetic systems of coercimeters with a specified field configuration.

(2) An informational model for the analysis of the magnetic-field strength distribution in axially symmetric MSs, which consist of field sources in the form of coils and FEs with nonlinear magnetic characteristics, was developed on the basis of the SIE method.

(3) Numerical experiments on the synthesis of coercimeters in the considered model examples made it possible to provide magnetic-field uniformity in the working volume at a level of 0.3%.

(4) The developed method allows synthesizing of MSs with nonuniform fields with large gradients for coercimeters based on the ponderomotive testing method.

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