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# Pareto-Optimal Parametric Synthesis of Axisymmetric Magnetic Systems with Allowance for Nonlinear Properties of the Ferromagnet

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**Abstract**—A method of Pareto-optimal synthesis is worked out for axisymmetric magnetic systems containing ferromagnetic elements with nonlinear magnetic properties. The method is based on the simultaneous application of the swarm intelligence paradigm and the evolution optimization with the help of genetic algorithms. Numerical examples of designing the devices for producing uniform and gradient magnetic fields in their working volume are considered.

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## INTRODUCTION

Optimal designing of magnetic systems containing ferromagnetic elements involves parametric synthesis in which the optimal values of structural parameters of the magnetic circuit, pole tips, and magnetizing coils are sought to ensure the required magnetic field distribution in the working volume of the device. The method used for calculating the field and the conditional optimization algorithm are equally important in this case.

In most cases, the magnetic field distribution given a priori can be obtained by via the search for the optimal shape of pole tips of the electromagnet. In [1], the finite-element method (FEM) was used for this purpose together with the local method for determining the extremum of the function being optimized and the sensitivity analysis method. In the general case, such an approach does not lead to the global solution to the optimization problem and gives only one local solution. In addition, the application of the FEM presumes the splitting into finite elements of not only the ferromagnet, but also the space surrounding it, which increases the dimensionality of the problem by an order of magnitude and requires an artificial limitation of the computational domain. In [2], the apparatus of spatial integral equations presuming discretization of only ferromagnetic elements of the magnetic system was employed for calculating the field; however, the local method of large-scale search for the extremum was also used for determining the optimal shape of pole tips.

In the synthesis of magnetic devices, the algorithms with global search properties appear as most promising. These algorithms include the simulated annealing (SA), genetic algorithms (GA), and particle swarm optimization (PSO) [3–5]. Since considerable computer time expenditures are required even for obtaining a single solution to the direct problem taking into account nonlinear magnetic characteristics of a ferromagnet, it is especially important that the optimization algorithm be able to find the global solution with the minimal number of computations for the target function. It was shown in [6] that simultaneous use of the PSO algorithm and GA makes it possible to elevate the convergence rate of the optimization process for various classes of functions (including multiextremum and ravine functions), which are often encountered in solving inverse incorrectly formulated problems (including those associated with synthesis of magnetic devices). The hybrid algorithm of particle swarm optimization with the evolution formation of the population composition developed in [6] differs from the available algorithms which employ simultaneously the PSO and GA search strategies [7, 8] in a more harmonious combination of both algorithms, the application of the crossover operator not only for positions of particles, but also for their velocities, as well as in the evolution of bonds between swarm particles.

In the synthesis of real magnetic systems containing ferromagnetic elements, it is necessary to take into account not only the requirements imposed on the magnetic field distribution, but also a number of additional factors such as the minimization of the mass of the device being designed, the reduction of the size of the magnetic system, the power consumed by the coil, and so on. Thus, it is necessary to solve the optimization problem in the multicriterion formulation. In [9–13], the aggregated vectorial criterion was used for this purpose, which made it possible to simultaneously take into account several requirements to the magnetic system being designed and to reduce the multicriterion problem of optimization to a single-criterion problem. The main feature of this approach is the choice of weight factors for individual criteria and setting their priorities.

Another problem is the necessity of taking into account the limitations imposed on the structural parameters of the magnetic system, which is most often solved by using penalty functions, which in turn requires a rational choice of numerical values of the coefficients and the penalty function and usually facilitates the ravines of the target function. An alternative approach to multicriterion optimization is the search for the Pareto set [14] that contains all solutions not dominated by other solutions. To single out nondominated solutions, it is convenient to use specially computed ranks. However, the problem of comparison of several solutions with the same values of ranks arises in this case.

This study aims at the development of a hybrid algorithm of the Pareto-optimal synthesis of axisymmetric magnetic systems with ferromagnetic elements possessing nonlinear properties and at the demonstration of its potentialities by considering some model examples.

## 1. METHOD OF PARETO-OPTIMAL SYNTHESIS

The solution of the problem of optimal designing of magnetic systems containing ferromagnetic structural elements involves the solution of two main subproblems, viz., the effective calculation of the field in the working volume of the magnetic system being designed and the development of the multicriterion optimization algorithm that makes it possible to determine the optimal values of structural parameters of the magnetic system being designed taking into account a number of additional criteria that are not connected directly with the requirements to the magnetic field configuration, but which may considerably affect the cost of manufacturing or operation of the device, its size, mass, etc. A high efficiency of the computational process of the synthesis can be attained by harmonizing the algorithms for computing the field and optimization taking into account the maximal number of specific features of the problem being solved. In terms of the concept used here, the solution of the optimal synthesis problem presumes multiple calculation of the field in the working volume for different values of structural parameters of the magnetic system. The speed of field computation can be increased by singling out computational procedures that give results independent to the maximal extent of the current value of structural parameters of the magnetic system into an individual block, which operates at the preliminary stage and the results are stored in the random access memory of the computer to be used at a later stage. In turn, the optimization algorithm forming the basis of the synthesis method must ensure the search for the global optimum of multiextremum and multidimensional ravine functions as well as the functions with regions of the plateau type. Since the values of the target functions are determined algorithmically (i.e., as a result of numerical calculation of the field), the application of the algorithms, in which the values of the derivatives of the target functions are used, is undesirable.

It is convenient to represent the structural parameters of the magnetic system being designed in the form of vector $\mathbf{x} = (x_1, x_2, ..., x_n)$ assuming the values from a certain subset $\mathbf{D} \subset \mathbf{R}^n$ defined by the set of constraints associated with the magnetic system geometry and physical performability. The requirements imposed on the magnetic system are specified in the form of the vectorial criterion $\mathbf{f}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_m(\mathbf{x})\}$. On the set of possible values of the vectorial criterion, the partial ordering relation known as the Pareto dominance is introduced, for which $\mathbf{f}^{(1)} \prec \mathbf{f}^{(2)}$ when the condition $f_i^{(1)} \le f_i^{(2)}$ is satisfied for all $i = \overline{1, m}$, while the condition $f_i^{(1)} < f_i^{(2)}$ holds for one value of $i$. This rule makes it possible to introduce the dominance relation on set $\mathbf{D}$, for which $\mathbf{x}_1 \prec \mathbf{x}_2$ when $\mathbf{f}(\mathbf{x}_1) \prec \mathbf{f}(\mathbf{x}_2)$. The solution $\mathbf{x}_1 \in \mathbf{D}$ is referred to as Pareto-optimal if the solution $\mathbf{x}_2 \in \mathbf{D}$ for which the condition $\mathbf{x}_1 \prec \mathbf{x}_2$ does not exists. The set of Pareto-optimal solutions is called the Pareto set, which will be denoted as $\mathbf{P}$, $\mathbf{P} \subset \mathbf{D}$. It should be noted that the elements of set $\mathbf{P}$ are pairwise incomparable.

For finding the Pareto-optimal values of structural parameters of the magnetic system being designed, we used the swarm optimization algorithm with the evolutionary formation of the swarm composition, which was developed earlier [6]. With the help of this algorithm, a certain subset of points $\mathbf{X} = (x_1, x_2, ..., x_N)$ representing the swarm of particles is selected at random from set $\mathbf{D}$. All solutions from set $\mathbf{X}$ are assigned ranks from the set denoted by $\mathbf{r} = \{r_1, r_2, ..., r_N\}$ in accordance with the number of dominating elements. The particles of zero rank are certain approximations of the Pareto-optimal solutions from subset $\mathbf{P}$. Elements of subset $\mathbf{X}$ are assorted in increasing order of ranks; elements of the same rank are compared in accordance with an additionally introduced aggregated criterion of the form $f = f_1^{\lambda_1} f_2^{\lambda_2} \dots f_m^{\lambda_m}$, where $\lambda_1, \lambda_2, ..., \lambda_m$ are real-valued exponents determining the priority of the criteria. The introduction of additional information in the form of an aggregated criterion makes it possible to indicate explicitly the priority in the choice of the solution among single-rank versions. Thus, particles with a lower rank and simultaneously the lower value of the aggregated criterion are characterized by a lower value of the index. In set $\mathbf{X}$, we fix two subsets $\mathbf{X}_{\mathrm{PSO}}$ and $\mathbf{X}_{\mathrm{MP}}$ with the number of elements $N_{\mathrm{PSO}}$ and $N_{\mathrm{MP}}$, respectively, which are connected by the relation $\mathbf{X}_{\mathrm{MP}} \subset \mathbf{X}_{\mathrm{PSO}} \subset \mathbf{X}$. Elements with the smallest indices fall into subsets $\mathbf{X}_{MP} \subset X_{PSO}$; i.e., $\mathbf{X}$ acquires the structure

$$\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{N_{MP}}, \mathbf{x}_{N_{MP+1}}, \ldots, \mathbf{x}_{N_{PSO}}, \mathbf{x}_{N_{PSO}+1}, \ldots, \mathbf{x}_N\}. \quad (1)$$

To subset $\mathbf{X}_{PSO}$, we apply a certain number of iterations of the PSO algorithm. For this purpose, the particles from the swarm are associated with their velocities (whose set is denoted by $\mathbf{V} = (\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_N)$ possessing an analogous structure.

At each iteration of the PSO algorithm, the positions and velocities of the particles from subset $\mathbf{X}_{PSO}$ are updated in accordance with the formulas

$$\mathbf{v}_i' = \omega\mathbf{v}_i + c_1\mathbf{r}_1(\mathbf{x}_i^p - \mathbf{x}_i) + c_2\mathbf{r}_2(\mathbf{x}_i^g - \mathbf{x}_i), \quad (2)$$

$$\mathbf{x}_i' = \mathbf{x}_i + \mathbf{v}_i', \quad (3)$$

where $i = \overline{1, N_{PSO}}$, $\mathbf{x}_i$ is the vector of the parameters characterizing the position of the $i$th particle, $\mathbf{v}_i$ is the velocity of the $i$th particle, $\omega$ is the inertial coefficient, $c_i$ is the coefficient characterizing the individual behavior of the $i$th particle, $\mathbf{x}_i^p$ is the best solution determined by the $i$th particle on its entire trajectory, $c_2$ is the coefficient characterizing the collective behavior of a particle, $\mathbf{x}_i^g$ is the best solution determined by the particles adjacent to the $i$th particle in accordance with a certain list of its neighbors, and $\mathbf{r}_1$ and $\mathbf{r}_2$ are the vectors containing random numbers with a uniform distribution law. If a particle leaves the boundaries of set $\mathbf{D}$, it returns to the nearest boundary with zero values of the corresponding velocity components. If a better solution is not obtained after several iterations, the bonds between the particles should be updated.

After a preset number of iterations of the PSO algorithm, the elements of set $\mathbf{X}$ which are not contained in subset $\mathbf{X}_{PSO}$ are replaced by the particles obtained as a result of application of genetic operators to elements of subset $\mathbf{X}_{MP}$. Genetic operators include binary stochastic operators known as crossovers, as well as unary operators known as mutations. The application of genetic operators affecting the positions of particles, their velocities, and lists of neighbors should be considered separately.

As examples of such generic operators, we can mention crossovers acting in accordance with the following schemes:

$$\mathbf{x}_{i_3} = \alpha\mathbf{x}_{i_1} + (1-\alpha)\mathbf{x}_{i_2}, \quad (4)$$

$$\mathbf{v}_{i_3} = \beta\mathbf{x}_{i_1} + (1-\beta)\mathbf{v}_{i_2}, \quad (5)$$

where $i_1$ and $i_2 = \overline{1, N_{MP}}$ are the indices of particles from $\mathbf{X}_{PM}$ randomly selected for crossing and referred to as parents, $i_3 = \overline{N_{PSO} + 1, N}$ are the indices of particles from $\mathbf{X}\backslash\mathbf{X}_{PSO}$ referred to as issues, and $\alpha$ and $\beta$ are random numbers with a uniform distribution law, which are generated so that the result of application of the crossovers being described, lies within the set of admissible solutions $\mathbf{D}$. In crossovers affecting the list of neighbors, the issues inherit the positions and velocities of their parents without changes, but their neighbors are randomly selected from the lists of neighbors of both parents. Accordingly, during mutations, the positions, velocities, and lists of neighbors of particles are perturbed at random in accordance with certain laws.

After realization of the above procedures, ranking and sorting of elements of set $\mathbf{X}$ are repeated. It should be noted that during sorting, the velocities, lists of neighboring particles, and best individual and collective solutions associated with particles are displaced together with the particles. To find an admissible solution, the operation stages of the optimization algorithm described above should be repeated in cyclic order.

## 2. METHOD OF ANALYSIS OF MAGNETIC FIELD DISTRIBUTION

To calculate the field produced by the magnetic system containing ferromagnetic elements with susceptibility $\chi(H)$, it is convenient to use the 3D integral equation written in the magnetization distribution [13],

$$\mathbf{H}(Q) + \mathbf{M}(Q) = \frac{1}{4\pi}\mathrm{rot}_Q \iiint_V \mathbf{M}(P)\mathrm{grad}_P \frac{1}{r_{PQ}} dV_P + \mathbf{H}^{(0)}(Q), \quad (6)$$

and supplemented with the constitutive equation of the form

$$\mathbf{M}(Q) = \chi(H)\mathbf{H}(Q), \quad (7)$$

where $\mathbf{r}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P$ is the vector connecting the source point P and observation point Q belonging to the ferromagnetic body and $\mathbf{H}^{(0)}(Q)$ is the strength of the primary field produced by the currents in the coils.

For the known distribution of magnetization M over the entire volume $V$ of the ferromagnet, the magnetic field strength at an arbitrary point Q of the working volume can be calculated by the formula

$$\bar{H}(Q) = \frac{1}{4\pi}\mathrm{curl}_Q \iiint_V \frac{\mathbf{M}(P)\mathbf{r}_{PQ}}{r_{PQ}^3} dV_P + H^{(0)}(Q). \quad (8)$$

In the case of an axisymmetric magnetic system, using the cylindrical system of coordinates and dividing the ferromagnetic body into annular elements with a constant step along coordinate axes $\rho$ and $z$, we can reduce, following [13], the solution of initial integral equation (6) (under the assumption of the piecewise-continuous magnetization distribution in the discretes) to the solution of the system of nonlinear equations of the form

$$\mathbf{F}^{-1}(\mathbf{M}) + \mathbf{M} = \mathbf{A}\mathbf{M} + \mathbf{H}^{(0)}, \quad (9)$$

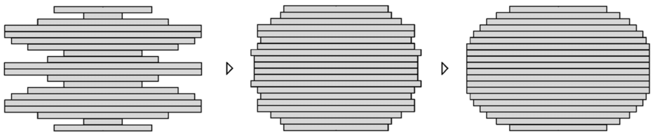

**Fig. 1.** Search for the shape of a uniformly magnetizable body.

where **A** is the matrix of the coefficients of mutual effect of annular discretes and $\mathbf{F}^{-1}(\mathbf{M})$ is the operator of the reciprocal magnetic characteristic. The most laborious stage in the solution of system of equations (9) is the evaluation of coefficients of matrix **A**. To avoid repeated calculations in the course of optimization, we propose that their values be calculated beforehand for all possible pairs of annular elements on a regular discrete grid in the possible space occupied by the ferromagnet [13] and then these values can be used whenever required. The allowance for translational symmetry along the $z$ axis has made it possible to considerably reduce the time required for computing the coefficients of matrix **A** and the random access memory of the computer because the values of the interference coefficients depend (correct to the sign) only on the mutual arrangement of a pair of annular elements (see [13] for details).

The system of nonlinear equations (9) was solved using the Newton–GMRES method, where the construction of the basis of the Krylov space, which is formally reduced to multiple multiplication of the matrix by a vector, takes most of the time in solving the system of linear equations. The use of some matrix transformations followed by rapid Fourier transformation for a convolution-type operation has made it possible to considerably accelerate this computational stage. A considerable gain in computer time with this approach can be obtained by reducing the discretization step in the solution search space.

## 3. VERIFICATION OF THE SYNTHESIS METHOD

The verification of the correctness of synthesis was carried out in the search for the shape of the body that can be uniformly magnetized in a uniform magnetic field. It is well known that only the bodies in the shape of an ellipsoid of revolution possess this property. In verification, an axisymmetric ferromagnetic body was represented by a set of cylinders with fixed radii followed by their division into annular elements. It was necessary to find the numerical values of the lengths of these cylinders, for which the magnetization of the body formed by them, was uniform to the maximal possible extent and had the maximal possible value. In the optimization problem considered here, the values of the quantity reciprocal to the magnetization at the center of the body and volume-averaged relative deviations of the radial and axial magnetization components played the role of particular minimization criteria.

To compare the solutions of the same rank, we used the objective function

$$f = \frac{\sum_{i=1}^{n} |\mathbf{M}_i - \mathbf{M}_0|^2 V_i}{|\mathbf{M}_0|^2 V}, \tag{10}$$

where $\mathbf{M}_i$ is the magnetization in the $i$th annular discrete, $\mathbf{M}_0$ is the magnetization at the center of the body, $V_i$ is the volume of the $i$th discrete, and $V$ is the total volume occupied by the ferromagnet.

The number of cylindrical elements was 10 and the radius and the maximal admissible length of the cylinders were 10 and 80 mm, respectively. Discretization of the cylinders was performed with a step of 0.5 mm. In numerical simulation, the external magnetic field strength was set at $H_0 = 1$ kA/m. Figure 1 shows the dynamics of variation of the shape of the body during synthesis. As a result of optimization, the deviation from uniformity of magnetization of the body was reduced to 10%. Upon an increase in the number of cylindrical elements, a tendency to further improvement of the uniformity of magnetization was observed.

Detailed information on the examples of verification of the Pareto-optimal method developed for bodies with known analytic solutions for determining the magnetic field distribution can be found in [13].

## 4. EXAMPLES OF NUMERICAL SIMULATION

### *4.1. Synthesis of Uniform Magnetic field*

As the first example of application of the method developed here, let us consider the synthesis of an axisymmetric magnetic system intended for producing a magnetic field with a high degree of uniformity in the working volume. The pole pieces were represented by a set of three cylindrical ferromagnetic elements. The parameters being varied were radii $\rho_i$ and heights $h_i$ of these elements, $i = \overline{1, 3}$, as well as the inner ($R_1$) and outer ($R_2$) radii of the coils, their length $h_c$, current density $j$ in them, wall thickness $d$, length $L$, and outer radius $R$ of the magnetic core (Fig. 2). The following constraints were imposed on the main geometrical parameters of the magnetic system expressed in millimeters:

$$\begin{cases} 5 \le d \le 10, \\ 60 \le L \le 120, \quad 20 \le R \le 40, \\ 0 \le \rho_1 \le \rho_2 \le \rho_3 \le R - d, \\ 20 \le h_i \le L/2 - 10. \end{cases} \tag{11}$$

Parameters $h_c$, $R_1$, and $R_2$ of the coils were varied within the limits ensuring their nonintersection with the magnetic core and the poles; current density $j$ was

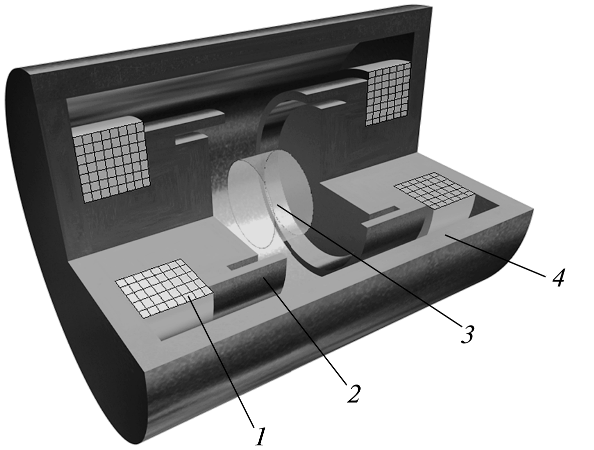


**Fig. 2.** Synthesized magnetic system ensuring a uniform magnetic field in the working volume: *1*—coils; *2*—pole elements; *3*—working volume; and *4*—magnetic core.

varied from 1 to 5 A/mm$^2$. The working volume had the shape of a cylinder with a radius and length of 10 mm; control points $k$ were located in this cylinder regularly with a step of 1 mm along the coordinate axes $\rho$ and $z$. For approximating the ferromagnetic elements in computational domain $R \times L$, a regular grid with a step of 1 mm along both axes was introduced. The total number of discretes of the system exceeded 3000.

The role of the partial criteria was played by the functions

$$\varphi_1 = \frac{1}{H_z^{(0)}}, \quad \varphi_2 = \frac{\max}{k}\left|\frac{\Delta H^{(k)}}{H^{(0)}}\right|, \quad\quad (12)$$
$$\varphi_3 = V, \quad \varphi_4 = j^2 V_c,$$

where $H^{(0)}$ is the magnetic field strength at the center of the working volume, $\Delta H^{(k)}$ is the absolute deviation of the magnetic field strength at the control points from its value at the center of the system, $V$ is the volume occupied by the ferromagnet, and $V_c$ is the volume occupied by the coils. Minimization of these criteria makes it possible to maximize the field strength at the center of the working volume, to ensure a high degree of the magnetic field uniformity, to minimize the volume occupied by the ferromagnet, and to reduce the power consumed by the coils.

To compare the solutions of different ranks, we used an aggregated criterion of the type

$$f = f_1^{\lambda_1} f_2^{\lambda_2} f_3^{\lambda_3}, \quad\quad (13)$$

where

$$f_1 = \sum_{k=1}^{K}\left(\frac{\Delta H^{(k)}}{H^{(0)}}\right)^2, \quad f_2 = \frac{V - V_{\min}}{V_{\max} - V_{\min}},$$
$$f_3 = \frac{P - P_{\min}}{P_{\max} - P_{\min}},$$

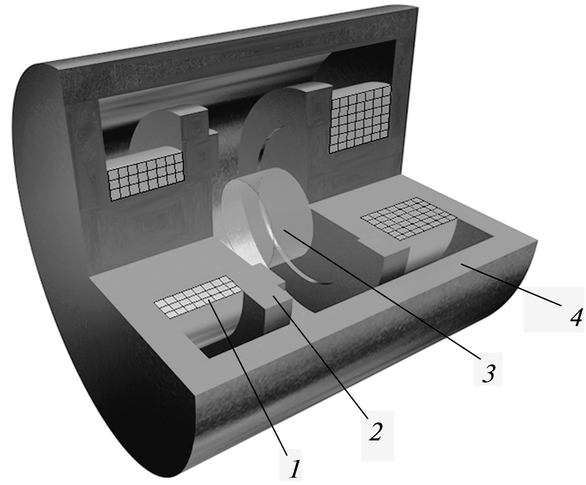


**Fig. 3.** Synthesized magnetic system ensuring the constancy of quantity grad $H_z$ in the working volume: *1*—coils; *2*—pole elements; *3*—working volume; and *4*—magnetic core.

$$P = j^2 V_c, \quad P_{\min} = j_{\min}^2 V_{c\,\min}, \quad P_{\max} = j_{\max}^2 V_{c\,\max},$$
$$\lambda_1 = 1, \quad \lambda_2 = \lambda_3 = 0.5.$$

The external appearance of the magnetic system is illustrated in Fig. 3. In optimization, mirror symmetry of the system relative to the $\rho Oz$ plane was assumed a priori. The optimal numerical values of the parameters of the magnetic system determined during synthesis are given in Table 1. Figure 4a shows the relative deviation of the magnetic field strength in the working volume from its value at the center of the magnetic system. The maximal relative error in the synthesized magnetic field distribution did not exceed 0.06%.

### *4.2. Synthesis of the Gradient Magnetic Field*

By way of another example, we consider the synthesis of an axisymmetric magnetic system intended for producing a field with a constant gradient grad $H_z$ in the working volume. The size of the working volume and the ranges of the structural parameters in this case were chosen the same as in the previous example, but the symmetry of the left and right halves of the magnetic system was not presumed a priori.

The first partial criterion whose minimization makes it possible to maximize the magnetic field gradient had the form

**Table 1.** Optimal values of structural parameters

| $d$ | $L$ | $R$ | $h_c$ | $R_1$ | $R_2$ | $j$, A/mm$^2$ | $\rho_1$ | $\rho_2$ | $\rho_3$ | $h_1$ | $h_2$ | $h_3$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| mm | | | | | | | mm | | | | | |
| 5 | 90 | 36 | 13 | 11 | 24 | 1.3 | 18 | 20 | 22 | 31 | 24 | 35 |

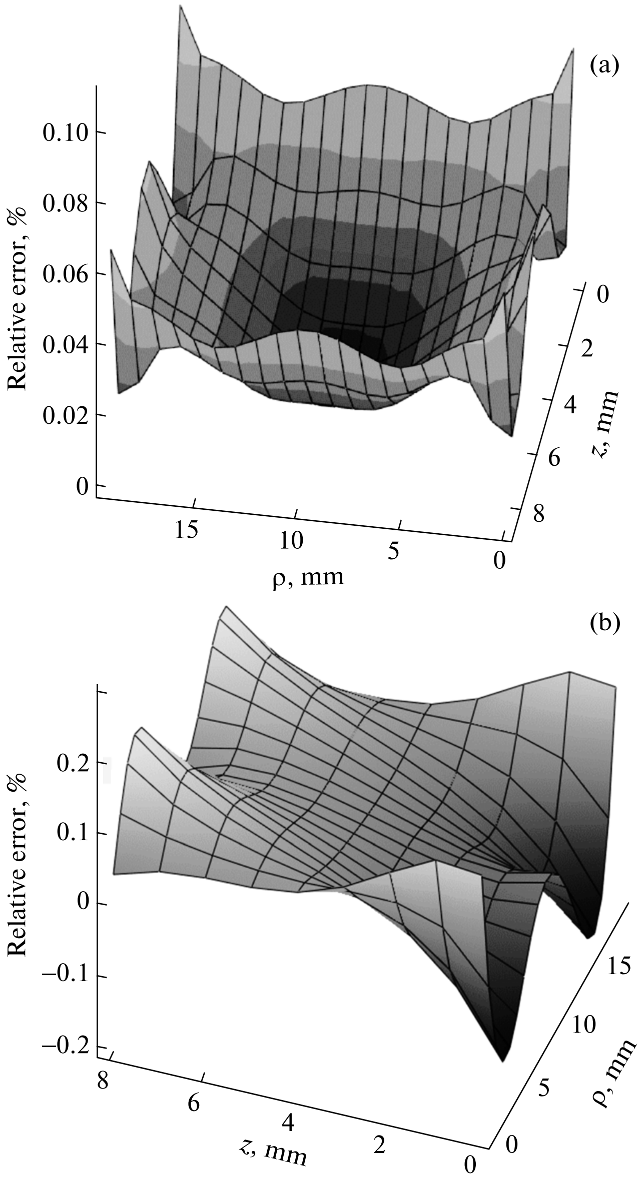


**Fig. 4.** Relative error in the synthesis of the magnetic field in the working volume of the magnetic system: (a) uniform distribution; (b) gradient distribution.

$$\varphi_1 = \left|\frac{H_{z2} - H_{z1}}{z_2 - z_1}\right|^{-1}, \tag{14}$$

where $z_1$ and $z_2$ are the coordinates of the points lying on the axis near the left and right ends of the working volume and $H_{z1}$ and $H_{z2}$ are the axial components of the magnetic field at these points. For grad $H_z$ = const, the magnetic field distribution along the $z$ axis obeys the equation

$$H_z = Az + B, \tag{15}$$

where

$$A = \frac{H_{z2} - H_{z1}}{z_2 - z_1}, \quad B = \frac{H_{z1}z_1 - H_{z2}z_2}{z_2 - z_1}.$$

To ensure the required field distribution in the working volume, we take for the second criterion the function

$$\varphi_2 = \frac{\max}{k}\left|\frac{H_z - H_z^k}{H_z}\right|. \tag{16}$$

The criteria taking into account the volume occupied by the ferromagnet and the power consumed by the coils were analogous to functions $\varphi_3$ and $\varphi_4$ from Eqs. (12). To compare the single-rank solutions, we used the aggregated criterion with

$$f_1 = \sum_{k=1}^{K}\left(\frac{H_z - H_z^k}{H_z}\right)^2 \tag{17}$$

and factor

$$f_4 = \frac{z_2 - z_1}{H_{z2} - H_{z1}}, \tag{18}$$

which makes it possible to maximize the magnetic field gradient.

The optimal numerical values of the main structural parameters of the core determined in the synthesis are $d = 6$ mm, $L = 80$ mm, and $R = 39$ mm. The values of the optimal parameters of the coils and pole elements of the magnetic system are given in Table 2.

Figure 3 shows the external appearance of the synthesized magnetic system. The relative deviation of the magnetic field strength in the working volume from the required distribution is shown in Fig. 4b. The error in the synthesis did not exceed 0.2%, and the attained value of the magnetic field gradient was 3.6 (kA/m)/cm.

**Table 2.** Optimal values of structural parameters

| Parameters of the magnetic system | $h_c$ | $R_1$ | $R_2$ | $j$, A/mm$^2$ | $\rho_1$ | $\rho_2$ | $\rho_3$ | $h_1$ | $h_2$ | $h_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | mm | | | | mm | | | | | |
| Left-hand part | 15 | 14 | 11 | 3.5 | 14 | 22 | 27 | 27 | 29 | 27 |
| Right-hand part | 17 | 20 | 23 | 3.6 | 6 | 18 | 26 | 29 | 29 | 30 |

## CONCLUSIONS

The proposed method for the synthesis can be used for optimal designing of a wide range of axisymmetric magnetic systems containing ferromagnetic elements with nonlinear magnetic properties. The hybrid algorithm of the Pareto-optimal synthesis makes it possible to find the optimal values of their structural parameters, while the accelerated method for solving the problem employing the fast Fourier transformation and taking into account the translational symmetry of the discretes considerably improves the efficiency of the synthesis process in the whole.

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*Translated by N. Wadhwa*

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