BULLETIN of Cherkasy State Technological University

Journal homepage: https://bulletin-chstu.com.ua/en

Vol. 30 No. 1. 2025

UDC 004.89:519.816
DOI: 10.62660/bcstu/1.2025.91

Information technology for solving the multi-criteria decision-making problem using the modified Fuzzy TOPSIS method

Anton Maksymov
PhD Student
Cherkasy State Technological University
18006, 460 Shevchenka Blvd, Cherkasy, Ukraine
https://orcid.org/0009-0002-0411-8164

Yurii Tryus*
Doctor of Pedagogical Sciences, PhD in Physical and Mathematical Sciences, Professor
Cherkasy State Technological University
18006, 460 Shevchenka Blvd, Cherkasy, Ukraine
https://orcid.org/0000-0002-0739-2065

Abstract. The relevance of the research topic is determined by the need to effectively solve multi-criteria decision-making problems in conditions of fuzzy information. In this regard, the creation of information technologies that would enable the user to select and use the most effective multi-criteria decision-making methods in conditions of fuzzy information is an important problem. The purpose of the study was to develop information technology for solving the multi-criteria decision-making problem using the modified Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) method based on the use of different metrics and the results of group expertise, which increases the reliability of the obtained decisions. Within the framework of the study, an analysis of the most popular multi-criteria decision-making methods, in particular, methods using the fuzzy set apparatus, was carried out. The article analyses different popular metrics for estimating the distances between a fuzzy positive ideal solution and a fuzzy negative ideal solution in the FTOPSIS method. A technique is proposed for comparing the results of applying different methods, in particular, FTOPSIS using triangular and trapezoidal fuzzy numbers, TOPSIS with triangular and trapezoidal fuzzy numbers for determining criteria weights, which makes it possible to analyse the scale of deviations between the obtained results and to assess the quality of the experts' work. The obtained results expand the possibilities of using TOPSIS and FTOPSIS methods for decision-making in conditions of multi-criteriality and uncertainty. As a practical application of the developed information technology and the modified FTOPSIS method, the article solves the problem of selecting the best of popular risk management standards in IT projects. This will increase the effectiveness of risk management in conditions of uncertainty and incompleteness of information, improve the validity of decisions made, as well as adapt the risk management process to specific conditions of each individual IT project

Keywords: MCDM; MADM; fuzzy sets; FTOPSIS; IT projects; risk management; risk management standards

INTRODUCTION

The relevance of multi-criteria decision-making problems lies in the fact that in real conditions of management, planning and analysis, it is necessary to take into account a set of conflicting criteria that affect

Article's History: Received: 13.10.2024; Revised: 24.02.2025; Accepted: 17.03.2025.

Suggested Citation:
Maksymov, A., & Tryus, Yu. (2025). Information technology for solving the multi-criteria decision-making problem using the modified Fuzzy TOPSIS method. *Bulletin of Cherkasy State Technological University*, 30(1), 91-106. doi: 10.62660/bcstu/1.2025.91.

*Corresponding author

the choice of the best solution. This is especially important in conditions of uncertainty, when it is necessary to find a compromise between different performance indicators, such as costs, quality, risks and system stability. Multi-criteria analysis methods make it possible not only to formalise the selection process, but also ensure the validity of decisions, increasing their effectiveness and objectivity. The use of modern methods and information technologies in multi-criteria decision-making helps to automate the decision-making process, which is a key factor in complex and dynamic systems. The scientific community is actively researching and developing new modifications of the FTOPSIS method. In particular, Y. Kustiyahningsih *et al.* (2024) in their work analysed the use of different types of fuzzy logic in the FTOPSIS method, but exclusively using the Euclidean metric to determine the distance between an ideal and an anti-ideal solution.

N. Rane & S. Choudhary (2023) in their study conducted a comparative analysis of the effectiveness of the Fuzzy AHP and FTOPSIS methods, which also used the classical Euclidean metric. The results showed that both methods were effective tools for multi-criteria decision-making under uncertainty, but FTOPSIS demonstrated a slightly higher sensitivity to changes in weight coefficients of the criteria compared to Fuzzy AHP. H. Yin *et al.* (2020) in their work used the classical Euclidean metric to measure the distance between alternatives in the context of multi-criteria analysis using the TOPSIS method. The authors drew attention to the limitations of the classical Euclidean distance, which may not always correctly reflect the difference between fuzzy or unstructured data, which often arises in practical problems. In this regard, they proposed a modification of the Euclidean distance, which was called the relative Euclidean distance.

In their study, F. Han *et al.* (2024) demonstrated the practical application of the FTOPSIS method using the Euclidean distance for solving multi-criteria decision-making problems. This study demonstrated the practical use of FTOPSIS, particularly in the context of using standard metrics in real decision-making problems. In the study of P. Talukdar & P. Dutta (2019), six different distance metrics were considered within the framework of the classical TOPSIS method, which allowed for a comparative analysis based on examples. However, this study needs to be expanded in the direction of formalising a general model of using different metrics for the FTOPSIS method. In turn, H.-S. Shyur & H.-S. Shih (2024) analysed the impact of the choice of metric on the results of ranking alternatives in the classical TOPSIS method. The authors emphasise the importance of the correct choice of metric, since different metrics can significantly change the ranking results, affecting decision-making in multi-criteria problems.

The study of H. Arman *et al.* (2022) was focused on the incorrect use of the Euclidean metric to measure the difference between fuzzy numbers. The researchers emphasised that although the Euclidean metric was used to measure the distance between points in multidimensional spaces, its application to fuzzy numbers was conceptually incorrect. The authors concluded that the use of the Euclidean metric in the context of fuzzy numbers can lead to inaccurate results, since such a metric does not reflect their essential difference. Summarising the results of the analysis of scientific sources, it can be stated that the vast majority of existing studies are focused on the use of the standard Euclidean metric in the TOPSIS method and its fuzzy modification – FTOPSIS.

Thus, one of the tasks of this study is to analyse the impact of the choice of metric on the results of the FTOPSIS method. The involvement of experts in assessing alternatives according to different criteria is an important aspect of using multi-criteria decision-making methods based on the fuzzy set apparatus. This leads to the fact that the obtained fuzzy assessments, especially during individual expertise, are subjective in nature. In this situation, to improve the accuracy of the assessment of alternatives, it is advisable to use an expertise involving several experts. In turn, this leads to the need to apply methods of aggregating expert opinions. So, there is a need to develop information technologies that provide group expertise using fuzzy multi-criteria decision-making methods and implement the procedure for aggregating expert assessments.

Therefore, an urgent problem arises, which consists in developing a modified FTOPSIS method using appropriate adapted metrics, as well as creating information technology that implements them and takes into account the results of group expertise for a more accurate and effective assessment and choice of the best alternative from many possible ones according to several utility criteria. It is the solution to this problem that this article was devoted to.

MATERIALS AND METHODS

The multi-criteria decision-making problem is to assess and select the best alternative from the set of $A = \{A_1, A_2, \dots, A_N\}$ possible options, taking into account the set of $C = \{C_1, C_2, \dots, C_M\}$ criteria that characterise these alternatives. Some criteria can be benefit criteria (the higher the value, the better), let's denote the set of these criteria as $C = \{C_1, C_2, \dots, C_M\}$, P is the number of benefit (maximisation) criteria ($0 \leq P \leq M$), and other criteria are cost criteria (the lower the value, the better), let's denote the set of these criteria as $C_{min} = \{C_{P+1}, C_{P+2}, \dots, C_M\}$, while the number of cost (minimisation) criteria is equal to M-P. Each C_j criterion has its own w_j weight, while $w_j > 0, \sum_{j=1}^{M} w_j = 1$. The effectiveness of each A_i $(i = \overline{1,N})$ alternative is assessed relative to each C_j $(j = \overline{1,M})$ criterion using the value of the corresponding f_j (A_i, w_j, x_{ij}) utility function, where x_{ij} – the assessment of the value of the C_j criterion relative to the A_i alternative taking into account the $w_j(i = \overline{1,N}, j = \overline{1,M})$ weight. Formally, the MCDM problem can be written as follows:

$$\begin{cases} \max\limits_{A_i \in A} f_j\left(A_i, w_j, x_{ij}\right), j = \overline{1, P}, \\ \min\limits_{A_i \in A} f_j\left(A_i w_j, x_{ij}\right), j = \overline{P+1, M}. \end{cases} \quad (1)$$

Then the $A^* \in A$ alternative, which satisfies the condition:

$$A^* = \begin{cases} \arg\max\limits_{A_i \in A} f_j\left(A_i, w_j, x_{ij}\right), j = \overline{1, P}, \\ \arg\min\limits_{A_i \in A} f_j\left(A_i, w_j, x_{ij}\right), j = \overline{P+1, M}. \end{cases} \quad (2)$$

is the solution to the problem (1).

The A^* alternative of the form (2) can be considered as an ideal solution to the problem (1), which in practice can only be found approximately.

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), proposed by C. Hwang & K. Yoon (1981), is one of the most well-known methods for an approximate solution to the multi-criteria decision-making problem of the form (1). This method is based on the concept that the chosen alternative should have the smallest distance to the positive ideal solution (PIS) (a solution that maximises benefit criteria and minimises cost criteria) and the largest distance to the negative ideal solution (NIS) (a solution that minimises benefit criteria and maximises cost criteria). Linguistic assessments are a convenient and effective tool for reflecting expert assessments, especially in cases where precise quantitative information is unavailable or uncertain. They allow experts to express their judgments in an understandable form, which is often based on natural language.

In their works, C. Chen (2000) and C. Chen *et al.* (2006) extended the TOPSIS method by a procedure that uses linguistic assessments in the form of triangular and trapezoidal fuzzy numbers (FNs). Let's consider some aspects of this approach. In general, the parametric form of a triangular fuzzy number is a triple (a, b, c), where a - left, b - middle, c - right parametres of this fuzzy number, and the parametric form of a trapezoidal fuzzy number is a quadruple (a, b, c, d), where a - left, b - left middle, c - right middle, d - right parametres of this fuzzy number. Examples of linguistic assessments and their representation in the form of triangular and trapezoidal fuzzy numbers are given in Table 1.

Table 1. Examples of linguistic assessments for fuzzy numbers

Linguistic terms	Fuzzy triangular numbers	Fuzzy trapezoidal numbers
Very low (VL)	(1, 1, 3)	(1, 1, 2, 3)
Low (L)	(1, 3, 5)	(2, 3, 4, 5)
Middle (M)	(3, 5, 7)	(4, 5, 6, 7)
High (H)	(5, 7, 9)	(6, 7, 8, 9)
Very high (VH)	(7, 9, 9)	(8, 9, 10, 10)

Source: created by the authors based on C. Chen (2000), C. Chen *et al.* (2006)

Linguistic assessments are linguistic terms to which certain fuzzy numbers correspond, usually in a triangular or trapezoidal parametric form (Fig. 1).

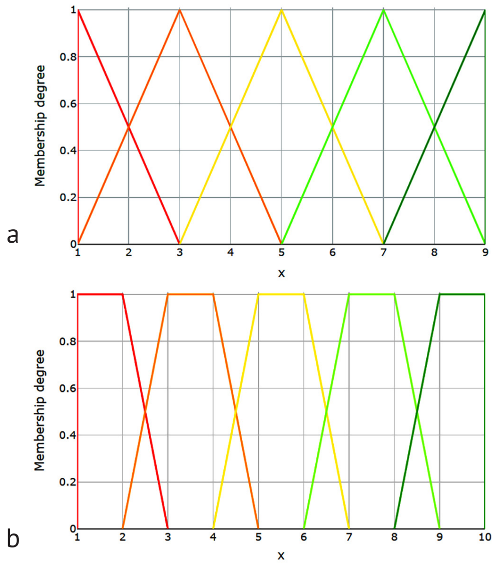

Figure 1. Examples of fuzzy triangular numbers (a) and trapezoidal numbers (b)

Source: created by the authors based on C. Chen *et al.* (2006)

To calculate the distance from each alternative to the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS) in FTOPSIS, C. Chen (2000) proposed a vertex method for calculating the distance between two fuzzy numbers, the essence of which is as follows. Let $\tilde{x}=(a_1,a_2,\ldots,a_n)$, $\tilde{y}=(b_1,b_2,\ldots,b_n)$ be fuzzy numbers specified by their parametres, where $n=3$ for triangular FNs and $n=4$ for trapezoidal FNs. Then the generalised adapted formula for calculating the Euclidean distance between triangular and trapezoidal FNs has the form:

$$d_E(\tilde{x}, \tilde{y}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(a_i - b_i)^2}. \quad (3)$$

Let's consider the features of calculating the distances between triangular and trapezoidal FNs using some popular metrics.

The Manhattan metric is a metric according to which the distance between two points $x=(x_1, x_2,\ldots, x_n)$ and $y=(y_1, y_2,\ldots, y_n)$ is equal to the sum of the moduli of the difference of their components (Rudin, 1987; Deza & Deza, 2016; Black, 2019):

$$d_{MH}(x, y) = \sum_{i=1}^{n} |x_i - y_i|. \quad (4)$$

Let $\tilde{x}=(a_1, a_2,\ldots, a_n)$, $\tilde{y}=(b_1, b_2,\ldots, b_n)$ be fuzzy numbers specified by their parametres, where $n = 4$ for triangular FNs and for trapezoidal FNs. Then the generalised adapted formula (4) for triangular and trapezoidal FNs has the form:

$$d_{MH}(\tilde{x},\tilde{y}) = \sum_{i=1}^{n}|a_i - b_i|, \quad (5)$$

where $n = 3$ for triangular FNs; $n = 4$ for trapezoidal FNs.

The Chebyshev metric is a metric that determines the distance between two -dimensional numerical vectors $x = (x_1, x_2,..., x_n)$ and $y = (y_1, y_2,..., y_n)$ as the maximum of the moduli of the difference of their components (Deza & Deza, 2016). The Chebyshev metric is often denoted by l_∞:

$$l_\infty(\vec{x},\vec{y}) = \max_{i=\overline{1,n}}|x_i - y_i|. \quad (6)$$

Then the generalised adapted formula (6) for FNs $\tilde{x} = (a_1, a_2,..., a_n)$, $\tilde{y} = (b_1, b_2,..., b_n)$ has the form:

$$d_\infty(\tilde{x},\tilde{y}) = \max_{1\le i\le n}|a_i - b_i|, \quad (7)$$

where $n = 3$ for triangular FNs; $n = 4$ for trapezoidal FNs.

The Minkowski metric is a parametric metric that can be considered as a generalisation of the Euclidean metric and the Manhattan metric. The Minkowski distance of p order between two points $x = (x_1, x_2,..., x_n)$ and $y = (y_1, y_2,..., y_n)$ is determined by the formula:

$$l_p(x,y) = \left(\sum_{i=1}^{n}|x_i - y_i|^p\right)^{\frac{1}{p}}. \quad (8)$$

The Minkowski metric is a generalisation of the Euclidean metric for the $p = 2$ parametre, and when $p = 1$, the Manhattan metric is obtained (Deza & Deza, 2016). If p goes to plus infinity, then the Minkowski distance approaches the Chebyshev distance:

$$\lim_{p\to+\infty}\left(\sum_{i=1}^{n}|x_i - y_i|^p\right)^{1/p} = \max_{i=\overline{1,n}}|x_i - y_i|. \quad (9)$$

Similarly, when p goes to minus infinity, the following formula is obtained:

$$\lim_{p\to-\infty}\left(\sum_{i=1}^{n}|x_i - y_i|^p\right)^{1/p} = \min_{i=\overline{1,n}}|x_i - y_i|. \quad (10)$$

The generalised adapted formula (8) for FNs $\tilde{x} = (a_1, a_2,..., a_n)$, $\tilde{y} = (b_1, b_2,..., b_n)$ has the form:

$$d_M(\tilde{x},\tilde{y}) = \left(\frac{1}{n}\sum_{i=1}^{n}|a_i - b_i|^p\right)^{\frac{1}{p}}, \quad (11)$$

where $n = 3$ for triangular FNs; $n = 4$ for trapezoidal FNs.

The Hamming metric is a metric that calculates the number of positions in which the corresponding symbols of two words of the same length are different. The Hamming distance between two points $x = (x_1, x_2,..., x_n)$ and $y = (y_1, y_2,..., y_n)$ is determined by the formula:

$$d_H(\tilde{x},\tilde{y}) = \sum_{i=1}^{n}|x_i - y_j|, \text{ where } |x_i - y_j| = \begin{cases}1, & x_i \ne y_j \\ 0, & x_i = y_j\end{cases}. \quad (12)$$

The generalised adapted formula (12) for FNs $\tilde{x} = (a_1, a_2,..., a_n)$, $\tilde{y} = (b_1, b_2,..., b_n)$ has the form:

$$d_H(\tilde{x},\tilde{y}) = \sum_{i=1}^{n}|a_i - b_i|, \text{ where } |a_i - b_i| = \begin{cases}1, & a_i \ne b_i \\ 0, & a_i = b_i\end{cases}, \quad (13)$$

where $n = 3$ for triangular FNs; $n = 4$ for trapezoidal FNs.

Let's consider the main steps of the method, which is a modification of the FTOPSIS method for the case of group expertise and uses different metrics for calculating the distance from each alternative to FPIS and FNIS in order to determine the best alternative.

Suppose that there is a group of decision-makers consisting of K experts.

Step 1. Assignment of fuzzy assessments to criteria and alternatives.

Each k-th expert of the $(k = \overline{1,K})$ group determines the linguistic assessment of the weight of each C_j $(j = \overline{1,M})$ criterion to which a triangular fuzzy number $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k)$ corresponds. Next, the k-th expert of the group determines the linguistic assessment for each A_i $(i = \overline{1,N})$ alternative with respect to the C_j $(j = \overline{1,M})$ criterion to which the triangular fuzzy number $\tilde{x}_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k)$ corresponds.

If fuzzy trapezoidal numbers $\tilde{w}_j^k$ and $\tilde{x}_{ij}^k$ correspond to linguistic assessments of the k-th expert of the group, then they have the following form, respectively:

$$\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k),\ \tilde{x}_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k). \quad (14)$$

Step 2. Calculation of aggregated fuzzy weights for criteria and aggregated fuzzy assessments for alternatives.

The aggregated fuzzy weight assessment in triangular form $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ for C_j criterion is calculated according to the formulas:

$$w_{j1} = \min_{k=\overline{1,K}}\{w_{j1}^k\},$$
$$w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{j2}^k\ ,\ w_{j3} = \max_{k=\overline{1,K}}\{w_{j3}^k\}, j = \overline{1,M}. \quad (15)$$

Similarly, the aggregated triangular fuzzy assessment $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ for A_i alternative with respect to C_j criterion is calculated according to the formulas:

$$a_{ij} = \min_{k=\overline{1,K}}\{a_{ij}^k\},$$
$$b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ij}^k\ ,\ c_{ij} = \max_{k=\overline{1,K}}\{c_{ij}^k\}, i = \overline{1,N},\ j = \overline{1,M}. \quad (16)$$

These formulas make it possible to obtain a common fuzzy assessment that takes into account minimum, average and maximum values from fuzzy assessments of all experts in the group.

Let $\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k)$ be the trapezoidal fuzzy weight assessment determined by the k-th expert in the group for C_j $(j = \overline{1,M})$ criterion. Then the aggregated fuzzy weight $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$ for C_j criterion can be calculated according to the following formulas:

$$\begin{gathered} w_{j1} = \min_{k=\overline{1,K}}\{w_{j1}^k\}\ ,\ w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{j2}^k\ , \\ w_{j3} = \frac{1}{K}\sum_{k=1}^{K} w_{j3}^k\ ,\ w_{j4} = \max_{k=\overline{1,K}}\{w_{j4}^k\},\ j = \overline{1,M} \end{gathered}. \quad (17)$$

Let $\tilde{x}_{ij}^k = \left(a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k\right)$ be the trapezoidal fuzzy assessment of the k-th expert of the decision-making group for A_i $(i = \overline{1, N})$ alternative with respect to C_j $(j = \overline{1, M})$ criterion. Then the aggregated fuzzy assessments for trapezoidal fuzzy numbers $\tilde{x}_{ij} = \left(a_{ij}, b_{ij}, c_{ij}, d_{ij}\right)$ are determined according to the formulas: $x\,\tilde{r}_{ij}$

$$a_{ij} = \min_{k=\overline{1,K}}\{a_{ij}^k\}, b_{ij} = \frac{1}{K}\sum_{k=1}^K b_{ij}^k, c_{ij} = \frac{1}{K}\sum_{k=1}^K c_{ij}^k,$$
$$d_{ij} = \max_{k=\overline{1,K}}\{d_{ij}^k\}, i = \overline{1, N}, j = \overline{1, M}. \quad (18)$$

Step 3. Calculation of the normalised fuzzy decision matrix.

The normalised fuzzy decision matrix is denoted as $\tilde{R} = [\tilde{r}_{ij}]$, in which for triangular FNs:

$$\tilde{r}_{ij} = \left(r_{ij1}, r_{ij2}, r_{ij3}\right) = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right),$$
$$i = \overline{1, N}, j = \overline{1, P}, \quad (19)$$

where $c_j^+ = \max_{i=\overline{1,N}}\{c_{ij}\}$ – for the benefit criterion, and

$$\tilde{r}_{ij} = \left(r_{ij1}, r_{ij2}, r_{ij3}\right) = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right),$$
$$i = \overline{1, N}, j = \overline{P+1, M}, \quad (20)$$

where $a_j^- = \min_{i=\overline{1,N}}\{a_{ij}\}$ - for the cost criterion.

In the normalised fuzzy matrix for trapezoidal FNs it will be:

$$\tilde{r}_{ij} = \left(r_{ij1}, r_{ij2}, r_{ij3}, r_{ij4}\right) = \left(\frac{a_{ij}}{d_j^+}, \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+}\right),$$
$$i = \overline{1, N}, j = \overline{1, P}, \quad (21)$$

where $d_j^+ = \max_{i=\overline{1,N}}\{d_{ij}\}$– for the benefit criterion, and

$$\tilde{r}_{ij} = \left(r_{ij1}, r_{ij2}, r_{ij3}, r_{ij4}\right) = \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right),$$
$$i = \overline{1, N}, j = \overline{P+1, M}, \quad (22)$$

where $a_j^- = \min_{i=\overline{1,N}}\{a_{ij}\}$ – for the cost criterion.

Step 4. Calculation of the weighted normalised fuzzy decision matrix.

Let's calculate the weighted normalised fuzzy decision matrix

$$\tilde{V} = \left(\tilde{v}_{ij}\right), i = \overline{1, N}, j = \overline{1, M}, \quad (23)$$

where by for triangular fuzzy assessments

$$\tilde{v}_{ij} = \left(v_{ij1}, v_{ij2}, v_{ij3}\right) =$$
$$= \left(r_{ij1} \cdot w_{j1}, r_{ij2} \cdot w_{j2}, r_{ij3} \cdot w_{j3}\right), \quad (24)$$

and for trapezoidal fuzzy assessments

$$\tilde{v}_{ij} = \left(v_{ij1}, v_{ij2}, v_{ij3}, v_{ij4}\right) =$$
$$= \left(r_{ij1} \cdot w_{j1}, r_{ij2} \cdot w_{j2}, r_{ij3} \cdot w_{j3}, r_{ij4} \cdot w_{j4}\right). \quad (25)$$

Step 5. Calculation of the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS).

The values of FPIS and FNIS are calculated according to the formulas:

$$V^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \cdots, \tilde{v}_M^+), \text{ where } \tilde{v}_j^+ = \max_{i=\overline{1,N}}\{v_{ijm}\}\ j = \overline{1, M}, \quad (26)$$

$$V^- = (\tilde{v}_1^-, \tilde{v}_2^-, \cdots, \tilde{v}_M^-), \text{ where } \tilde{v}_j^- = \min_{i=\overline{1,N}}\{v_{ij1}\}\ j = \overline{1, M}, \quad (27)$$

where $m = 3$ for triangular FNs; $m = {}^4 3$ for trapezoidal FNs.

Step 6. Calculation of distances from each alternative to FPIS and FNIS.

The distance from A_i alternative to FPIS d_i^+ and to FNIS d_i^- is calculated according to the formulas:

$$d_i^+ = \sum_{j=1}^M d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^+\right), \quad (28)$$

$$d_i^- = \sum_{j=1}^M d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^-\right), i = \overline{1, N}, \quad (29)$$

where $d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^+\right)$ and $d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^-\right)$ according to the user's choice are determined as follows:

- for the Euclidean metric, taking into account (3):

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) =$$
$$= d_E\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) = \sqrt{\frac{1}{n}\sum_{k=1}^n (v_{ijk} - v_{jk}^+)^2}, \quad (30)$$

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) =$$
$$= d_E\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) = \sqrt{\frac{1}{n}\sum_{k=1}^n (v_{ijk} - v_{jk}^-)^2}, \quad (31)$$

- for the Manhattan metric, taking into account (5):

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) =$$
$$= d_{MH}\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) = \sum_{k=1}^n |v_{ijk} - v_{jk}^+|, \quad (32)$$

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) =$$
$$= d_{MH}\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) = \sum_{k=1}^n |v_{ijk} - v_{jk}^-|, \quad (33)$$

- for the Chebyshev metric, taking into account (7):

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) =$$
$$= d_\infty\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) = \max_{k=\overline{1,n}}|v_{ijk} - v_{jk}^+|, \quad (34)$$

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) =$$
$$= d_\infty\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) = \max_{k=\overline{1,n}}|v_{ijk} - v_{jk}^-|, \quad (35)$$

- for the Minkowski metric, taking into account (11):

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) =$$
$$d_M\left(\tilde{v}_{ij}, \tilde{v}_j^+\right) = \left(\frac{1}{n}\sum_{k=1}^n |v_{ijk} - v_{jk}^+|^p\right)^{\frac{1}{p}}, \quad (36)$$

$$d_{metrics}\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) =$$
$$= d_M\left(\tilde{v}_{ij}, \tilde{v}_j^-\right) = \left(\frac{1}{n}\sum_{k=1}^n |v_{ijk} - v_{jk}^-|^p\right)^{\frac{1}{p}}, \quad (37)$$

at $p \geq 3$

- for the Hamming metric, taking into account (13):

$$d_{metrics}(\tilde{v}_{ij},\tilde{v}_j^+) = = d_H(\tilde{v}_{ij},\tilde{v}_j^+) = \sum_{k=1}^{n} |v_{ijk} - v_{jk}^+|, \quad (38)$$

$$d_{metrics}(\tilde{v}_{ij},\tilde{v}_j^-) = = d_H(\tilde{v}_{ij},\tilde{v}_j^-) = \sum_{k=1}^{n} |v_{ijk} - v_{jk}^-|, \quad (39)$$

where

$$|v_{ijk} - v_{jk}^+| = \begin{cases} 1, & v_{ijk} \neq v_{jk}^+ \\ 0, & v_{ijk} = v_{jk}^+ \end{cases}, \quad (40)$$

$$|v_{ijk} - v_{jk}^-| = \begin{cases} 1, & v_{ijk} \neq v_{jk}^- \\ 0, & v_{ijk} = v_{jk}^-, \end{cases} \quad (41)$$

$n = 3$ for triangular FNs and $n = 4$ for trapezoidal FNs.

Step 7. Calculation of the closeness coefficients to FPIS and FNIS for each alternative according to chosen metrics.

The CC_i^- closeness coefficient for each A_i alternative to the fuzzy negative ideal solution is calculated using the following formula:

$$CC_i^- = \frac{d_i^-}{d_i^- + d_i^+}, i = \overline{1,N}. \quad (42)$$

The CC_i^+ closeness coefficient for each A_i alternative to the fuzzy positive ideal solution is calculated using the following formula:

$$CC_i^+ = \frac{d_i^+}{d_i^- + d_i^+}, i = \overline{1,N}. \quad (43)$$

Step 8. Ranking of alternatives according to chosen metrics. In this step, for each chosen metric, the best A^* alternative is determined as the alternative that is furthest from FNIS, i.e. with the maximum CC_i^- closeness coefficient, and is defined as follows:

$$A^* = A_{i^*}, \quad (44)$$

where

$$i^* = \arg \max_{i=\overline{1,N}} \{CC_i^-\}, \quad (45)$$

or A^* is selected as the alternative that is closest to FPIS, i.e. with the smallest CC_i^+ closeness coefficient, and is defined as:

$$A^* = A_{i^*}, \quad (46)$$

where

$$i^* = \arg \min_{i=\overline{1,N}} \{CC_i^+\}. \quad (47)$$

Therefore, the larger the CC_i^- value, the closer the A_i alternative is to the fuzzy positive ideal solution V^+ and further from the fuzzy negative ideal solution V^-, or the smaller the CC_i^+ value, the closer the A_i alternative is to the fuzzy positive ideal solution V^+ and further from the fuzzy negative ideal solution V^-. The A_i alternatives are ordered according to decrease in CC_i^- values, or according to increase in CC_i^+ values, which makes it possible to determine the rank of each R_i $(i = \overline{1,N})$ alternative for chosen metrics. In this case, some alternatives may have the same ranks, including rank 1.

Step 9. Output of results. After calculating the ranks of alternatives according to chosen metrics, the results are presented in tabular form, the structure of which is given in Table 2, where R_{ji} – the rank of A_i $(i = \overline{1,N})$ alternative according to the $d_{metricsj}$ $j = \overline{1,m}$ metric, m – the number of chosen metrics.

Table 2. Output of the results of the modified FTOPSIS method

Metrics	Alternatives	d_i^+	d_i^-	CC_i^-	Rank of alternatives
$d_{metrics_1}$	A_1	d_{11}^+	d_{11}^-	CC_{11}^-	R_{11}
	A_2	d_{12}^+	d_{12}^-	CC_{12}^-	R_{12}
	…	…	…	…	…
	A_N	d_{1N}^+	d_{1N}^-	CC_{1N}^-	R_{1N}
$d_{metrics_2}$	A_1	d_{21}^+	d_{21}^-	CC_{21}^-	R_{21}
	A_2	d_{22}^+	d_{22}^-	CC_{22}^-	R_{22}
	…	…	…	…	…
	A_N	d_{2N}^+	d_{2N}^-	CC_{2N}^-	R_{2N}
…	…	…	…	…	…
$d_{metrics_m}$	A_1	d_{m1}^+	d_{m1}^-	CC_{m1}^-	R_{m1}
	A_2	d_{m2}^+	d_{m2}^-	CC_{m2}^-	R_{m2}
	…	…	…	…	…
	A_N	dm_N^+	dm_N^-	CC_{mN}^-	R_{mN}

Source: created by the authors

For the software implementation of the proposed modified Fuzzy TOPSIS method, the authors created a module that is an integral part of the web-oriented Decision Support System (DSS) "Decisioner" (n.d.). In the developed DSS, the PHP language is used to create queries to the MySQL database. This programming language is also used to distribute access between users depending on their role. The information and registration part of the DSS was created using the CMS WordPress. The LaTeX data markup language was used to display formulas in the theoretical information of the methods. The JavaScript language without frameworks was used to implement the decision-making methods in the DSS. Actually, in order for JavaScript and PHP to interact with the user, standard HTML5 and CSS3 tools were chosen for visual design. The JQuery library was used for the interaction of algorithms of the methods with the HTML code of the page. The amCharts and Plotly libraries were used to display graphs.

RESULTS AND DISCUSSION

Multi-Criteria Decision Making (MCDM) is divided into two main categories:

- Multi-Objective Decision Making (MODM) is an approach to optimisation, in which a solution is chosen from a set of possible alternatives by finding a compromise between several objective functions. This approach is widely used in linear and non-linear programming problems, in particular for optimisation of production processes, resource allocation and management of complex systems under conditions of multi-criteriality;
- Multi-Attribute Decision Making (MADM) is an approach focused on choosing the best alternative from a discrete set of solutions based on several attributes (criteria). This is useful for problems where it is necessary to assess and rank a finite number of options.

Therefore, MODM is usually used in optimisation problems, and MADM is used to assess and choose effective alternatives. Both approaches are important components of MCDM and are used depending on the specific problem and available input data.

The analysis of modern research, such as F. Han *et al.* (2024), shows that among MADM methods, considerable attention is paid to methods based on fuzzy set theory, in particular the FTOPSIS method and the FAHP method. At the same time, the FTOPSIS method is more effective compared to the FAHP method in a number of situations due to several key aspects. FTOPSIS is convenient for the practical application, since it focuses on finding the most ideal solution that is as close as possible to the ideal (best) option and as far away as possible from the worst option. This makes the method easy to understand and apply, especially when it is necessary to easily and quickly assess alternatives according to several criteria. FAHP, in turn, often requires more computational steps and deeper analysis, including a hierarchy of criteria and sub-criteria, which can be more difficult in the case of a large number of criteria.

FTOPSIS works well with large numbers of alternatives and criteria. This allows for rapid assessment within large data sets. FTOPSIS does not require the construction of a complex hierarchy of criteria, which simplifies the decision-making process, especially in cases where complex relationships between criteria are not obvious. FAHP can become difficult with large numbers of criteria and/or alternatives, as it requires the construction of a hierarchical structure and the comparison of each pair of criteria according to the main criterion, as well as the comparison of each pair of alternatives according to each criterion, which quickly increases the amount of calculations. FAHP requires careful construction of the hierarchy structure of the research object and important comparisons between each level of the hierarchy, which can be difficult and time-consuming. In general, FTOPSIS is effective in cases, where it is necessary to quickly and efficiently assess alternatives according to a large number of criteria.

Most modern approaches to assessing distances in TOPSIS and FTOPSIS methods (Awasthi *et al.*, 2011; Arman *et al.*, 2022) are usually based on the Euclidean metric. However, in the context of complex and multidimensional data, this metric does not always adequately reflect the real differences between alternatives. This emphasises the need to study and implement alternative metrics, such as the Manhattan metric, Chebyshev, Minkowski and Hamming metrics. However, the use of alternative metrics in the context of fuzzy sets requires additional adaptation. Most classical metrics were developed to work with precise numerical data and do not take into account the peculiarities of representing fuzzy data, for example, in the form of triangular or trapezoidal fuzzy numbers. This complicates the direct use of standard formulas for calculating distances between alternative assessments. Therefore, it is necessary to develop new metrics or modify existing metrics so that they can correctly process fuzzy data, ensuring accuracy and reliability of the assessment. The study proposes a modified FTOPSIS method, which provides an opportunity to find effective alternatives according to the most popular metrics adapted to calculations with fuzzy numbers.

Practical application of the developed information technology and the modified Fuzzy TOPSIS method is demonstrated on the problem of choosing a risk management standard in IT projects based on their analysis, given in the author's article of A. Maksymov (2025). Let the decision makers need to determine the most effective risk management standard to be used by the IT company from the following alternatives: A_1 – use of ISO 31000:2018 (2018); A_2 – use of NIST 800-53 (National Institute of Standards and Technology Special Publication 800-53, 2020); A_3 – use of the PMBOK guide (2021). In this case, it is proposed to be guided by the following C_j criteria for assessing the effectiveness of choosing a risk management standard ($j = \overline{1,8}$):

- C_1 – number of countries implementing the standard (measured in quantity and to be maximised);
- C_2 – impact on the number of incidents/violations (measured in percentage and to be maximised);
- C_3 – impact on productivity (measured in percentage and to be maximised);
- C_4 – impact on employee satisfaction level (measured in percentage and to be maximised);

- C_5 – impact on financial performance (measured in percentage and to be maximised);
- C_6 – implementation costs (measured in thousands of $ and to be minimised);
- C_7 – implementation time (measured in months and to be minimised);
- C_8 – number of training hours for personnel (measured in hours and to be minimised).

For example, the results of an individual expertise of the problem are given. Based on the authors' analysis of risk management standards, linguistic assessments of the weights of the C_j $(j = \overline{1,8})$ criteria were determined, as well as for each A_i $(i = \overline{1,3})$ alternative according to each C_j criterion, both explicit values and intervals of utility, and their linguistic assessments given in Table 3 were determined.

Table 3. Assessments of different risk management standards for IT projects according to the specified criteria

	C_1 (countries)	C_2 (%)	C_3 (%)	C_4 (%)	C_5 (%)	C_6 (k$)	C_7 (months)	C_8 (hours)
Weight	Medium (M)	High (H)	High (H)	High (H)	Very High (VH)	Very High (VH)	High (H)	Medium (M)
min/max	max	max	max	max	max	min	min	min
A_1	100+ Very High (VH)	10-30 Medium (M)	10-20 High (H)	75-85 High (H)	10-15 Medium (M)	5-20 Medium (M)	3-6 Medium (M)	20-40 Medium (M)
A_2	50+ High (H)	20-40 High (H)	15-25 Very High (VH)	70-80 High (H)	20-30 Very High (VH)	50-150 Very High (VH)	6-12 High (H)	40-80 High (H)
A_3	100+ Very High (VH)	30-50 High (H)	20-30 Very High (VH)	80-90 High (H)	15-20 High (H)	20-100 High (H)	6-12 High (H)	40-60 High (H)

Source: created by the authors

To determine the best alternative among the risk management standards for IT projects according to certain criteria, calculations were first carried out using the modified FTOPSIS method by the appropriate *DSS "Decisioner"* module. Figure 2 demonstrates the first step in solving the problem, in particular, filling in the appropriate fields of the module with information about the purpose of the problem and its structural components: alternatives and criteria.

In the next step, it is necessary to select metrics and the type of fuzzy numbers for calculation, for example, as shown in Figure 3.

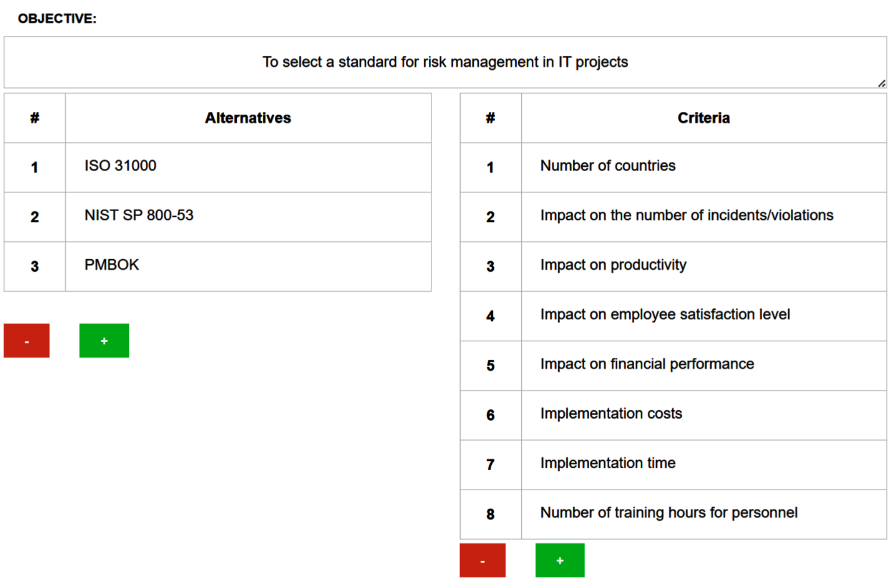

Figure 2. Setting the goal, alternatives and criteria in the DSS "Decisioner"

Source: created by the authors

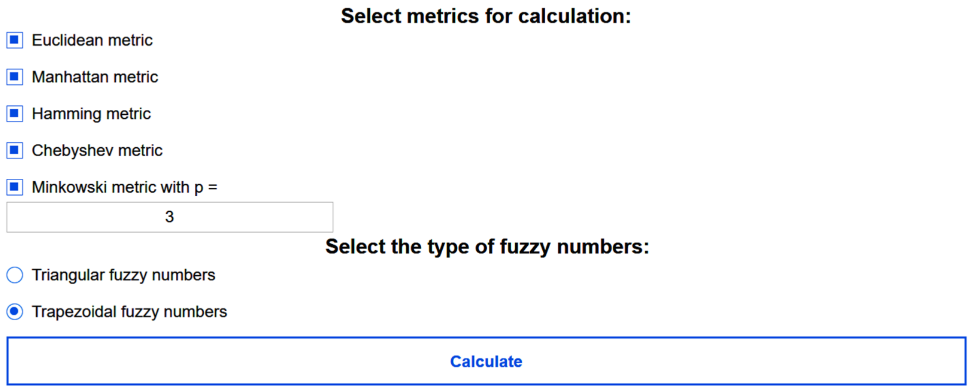

Figure 3. Selection of metrics and type of fuzzy numbers for calculation
Source: created by the authors

In the next step, the expert needs to fill in the initial matrix of assessments in the format of fuzzy linguistic assessments, for example, based on the data in Table 1 (Fig. 4).

	Number of countries	Impact on the number of incidents/violations	Impact on productivity	Impact on employee satisfaction level	Impact on financial performance	Implementation costs	Implementation time	Number of training hours for personnel
min/max	max	max	max	max	max	min	min	min
Weight	M	H	H	H	VH	VH	H	M
ISO 31000	VH	M	H	H	M	M	M	M
NIST SP 800-53	H	H	VH	H	VH	VH	H	H
PMBOK	VH	H	VH	H	H	H	H	H

Figure 4. Filling in the initial matrix of assessments in the format of linguistic terms
Source: created by the authors

After confirming the input data, the DSS automatically calculates the normalised fuzzy decision matrix $\tilde{R}=[\tilde{r}_{ij}]$ of the form (19)-(20) for triangular FNs, or of the form (21)-(22) for trapezoidal FNs (Fig. 5).

	C1	C2	C3	C4	C5	C6	C7	C8
min/max	**max**	**max**	**max**	**max**	**max**	**min**	**min**	**min**
ISO 31000	0.8000 0.9000 1.0000 1.0000	0.4444 0.5556 0.6667 0.7778	0.6000 0.7000 0.8000 0.9000	0.6667 0.7778 0.8889 1.0000	0.4000 0.5000 0.6000 0.7000	0.5714 0.6667 0.8000 1.0000	0.5714 0.6667 0.8000 1.0000	0.5714 0.6667 0.8000 1.0000
NIST SP 800-53	0.6000 0.7000 0.8000 0.9000	0.6667 0.7778 0.8889 1.0000	0.8000 0.9000 1.0000 1.0000	0.6667 0.7778 0.8889 1.0000	0.8000 0.9000 1.0000 1.0000	0.4000 0.4000 0.4444 0.5000	0.4444 0.5000 0.5714 0.6667	0.4444 0.5000 0.5714 0.6667
PMBOK	0.8000 0.9000 1.0000 1.0000	0.6667 0.7778 0.8889 1.0000	0.8000 0.9000 1.0000 1.0000	0.6667 0.7778 0.8889 1.0000	0.6000 0.7000 0.8000 0.9000	0.4444 0.5000 0.5714 0.6667	0.4444 0.5000 0.5714 0.6667	0.4444 0.5000 0.5714 0.6667

Figure 5. Normalised fuzzy decision matrix for trapezoidal FNs
Source: created by the authors

In the next step, the DSS automatically calculates the weighted normalised fuzzy decision matrix $\tilde{V} = [\tilde{v}_{ij}]$ of the form (23)-(25) (Fig. 6).

Figure 7 shows the result of calculating the best A^* alternative with a display of the distances from each A_i alternative to FPIS (d_i^+) of the form (28) and FNIS (d_i^-) of the form (29) for selected metrics, as well as the CC_i^- closeness coefficients of the form (42) ($i = \overline{1,3}$) and R_{ji} – the rank of A_i ($i = \overline{1,N}$) alternative according to the $d_{metrics_j}$ ($j = \overline{1,m}$) metric, m – the number of selected metrics.

	C1 (max)	C2 (max)	C3 (max)	C4 (max)	C5 (max)	C6 (min)	C7 (min)	C8 (min)
A1	3.2000 4.5000 6.0000 7.0000	2.6667 3.8889 5.3333 7.0000	3.6000 4.9000 6.4000 8.1000	4.0000 5.4444 7.1111 9.0000	3.2000 4.5000 6.0000 7.0000	4.5714 6.0000 8.0000 10.0000	3.4286 4.6667 6.4000 9.0000	2.2857 3.3333 4.8000 7.0000
A2	2.4000 3.5000 4.8000 6.3000	4.0000 5.4444 7.1111 9.0000	4.8000 6.3000 8.0000 9.0000	4.0000 5.4444 7.1111 9.0000	6.4000 8.1000 10.0000 10.0000	3.2000 3.6000 4.4444 5.0000	2.6667 3.5000 4.5714 6.0000	1.7778 2.5000 3.4286 4.6667
A3	3.2000 4.5000 6.0000 7.0000	4.0000 5.4444 7.1111 9.0000	4.8000 6.3000 8.0000 9.0000	4.0000 5.4444 7.1111 9.0000	4.8000 6.3000 8.0000 9.0000	3.5556 4.5000 5.7143 6.6667	2.6667 3.5000 4.5714 6.0000	1.7778 2.5000 3.4286 4.6667
V+	3.2000 4.5000 6.0000 7.0000	4.0000 5.4444 7.1111 9.0000	4.8000 6.3000 8.0000 9.0000	4.0000 5.4444 7.1111 9.0000	6.4000 8.1000 10.0000 10.0000	4.5714 6.0000 8.0000 10.0000	3.4286 4.6667 6.4000 9.0000	2.2857 3.3333 4.8000 7.0000
V-	2.4000 3.5000 4.8000 6.3000	2.6667 3.8889 5.3333 7.0000	3.6000 4.9000 6.4000 8.1000	4.0000 5.4444 7.1111 9.0000	3.2000 4.5000 6.0000 7.0000	3.2000 3.6000 4.4444 5.0000	2.6667 3.5000 4.5714 6.0000	1.7778 2.5000 3.4286 4.6667

Figure 6. Weighted normalised fuzzy decision matrix for trapezoidal FNs

Source: created by the authors

Metrics	Alternatives	d_i^+	d_i^-	CC_i^-	Rank of alternatives
Euclidean metric	A1	7.4563	8.8192	0.5419	1
	A2	8.8192	7.4563	0.4581	3
	A3	8.2978	8.0161	0.4914	2
Manhattan metric	A1	8.5222	9.2767	0.5212	1
	A2	9.2767	8.5222	0.4788	3
	A3	8.7794	9.0196	0.5067	2
Hamming metric	A1	12.0000	16.0000	0.5714	1
	A2	16.0000	12.0000	0.4286	3
	A3	16.0000	20.0000	0.5556	2
Chebyshev metric	A1	2.5333	3.8444	0.6028	1
	A2	3.8444	2.5333	0.3972	3
	A3	3.5556	2.8222	0.4425	2
Minkowski metric with p = 3	A1	7.1764	9.0394	0.5574	1
	A2	9.0394	7.1764	0.4426	3
	A3	8.4769	7.7997	0.4792	2

Answer: The best alternative corresponds to the option with a ranking of "1"

Figure 7. Result of calculating the best alternative for trapezoidal FNs

Source: created by the authors

Based on the results obtained according to the FTOPSIS method for different metrics, ISO 31000:2018 (2018) (A_1 alternative) according to all selected metrics is the best option for the risk management standard for IT projects among the proposed alternatives. The results of the experiment on solving the problem with the help of different modifications of the FTOPSIS method and the classical TOPSIS method with linguistic assessments of criteria weights are shown below:

1) FT3FNs – FTOPSIS using triangular FNs (see Fig. 1 a);

2) FT4FNs – FTOPSIS using trapezoidal FNs (see Fig. 1 b);

3) TFW3FNs – TOPSIS with fuzzy triangular numbers for determining criteria weights;

4) TFW4FNs – TOPSIS with fuzzy trapezoidal numbers for determining criteria weights.

In the TFW3FNs, TFW4FNs methods, in order to simplify the expert assessment of w_j, $(j = \overline{1, M})$ criteria weights, which in sum must be equal to one, in the classical TOPSIS method, linguistic assessments of the weights of each C_j criterion to which fuzzy numbers $\widetilde{w}_j = (w_{j1}, w_{j2}, \dots, w_{jn})$, $(j = \overline{1, M})$, correspond, where $n = 3$ for triangular FNs and $n = 4$ for trapezoidal FNs, were used. The process of defuzzification of fuzzy numbers was carried out according to the following rule:

1. Finding of criteria weights as the average value of fuzzy assessment parametres.

For each $\widetilde{w}_j = (w_{j1}, w_{j2}, \dots, w_{jn})$ the arithmetic mean of its parametres is calculated:

$$w_j = \frac{w_{j1} + w_{j2} + \dots + w_{jn}}{n}, j = \overline{1, M}, \tag{48}$$

where $n = 3$ for triangular FNs and $n = 4$ for trapezoidal FNs.

2. Calculation of the sum of average values:

$$ws = \sum_{j=1}^{M} w_j. \tag{49}$$

3. Normalisation of average values of the weights. Each w_j average value is converted into a fraction of the total sum "ws" so that all normalised assessments as a result are in the [0,1] range and their sum is equal to 1:

$$wn_j = \frac{w_j}{ws}, j = \overline{1, M}. \tag{50}$$

Fig. 8 shows a table with problem input data for the TOPSIS method with linguistic assessments of criteria weights to which trapezoidal FNs correspond (see Table 1).

Fig. 9 shows a table with problem input data for the TOPSIS method with normalised assessments of criteria weights according to formulas (48)-(50) for $n = 4$.

	Number of countries	Impact on the number of incidents/violations	Impact on productivity	Impact on employee satisfaction level	Impact on financial performance	Implementation costs	Implementation time	Number of training hours for personnel
min/max	max	max	max	max	max	min	min	min
Weight	M	H	H	H	VH	VH	H	M

Figure 8. Problem input data for the TOPSIS method with linguistic assessments of criteria weights
Source: created by the authors

	Number of countries	Impact on the number of incidents/violations	Impact on productivity	Impact on employee satisfaction level	Impact on financial performance	Implementation costs	Implementation time	Number of training hours for personnel
min/max	max	max	max	max	max	min	min	min
Weight	0.09243	0.12605042016806	0.12605042	0.1260504	0.1554621	0.15546218487	0.126050420	0.092436
ISO 31000	100	20	15	80	12.5	12.5	4.5	30
NIST SP 800-53	50	30	20	75	25	100	9	60
PMBOK	100	40	25	85	17.5	60	9	50

Figure 9. Problem input data for the TOPSIS method with normalised assessments of criteria weights
Source: created by the authors

In order to visually display the differences between the results when using different metrics and methods, a matrix of deviations between the results obtained by different methods was calculated according to such technique. Let there be a set of $M_1, M_2, \ldots, M_n$ methods, each of which gives closeness coefficients of each A_i $(i = \overline{1,N})$ alternative according to different metrics in the form of a matrix:

$$E^q = \begin{pmatrix} CC_{11}^q & CC_{12}^q & \cdots & CC_{1m}^q \\ CC_{21}^q & CC_{22}^q & \cdots & CC_{2m}^q \\ \vdots & \vdots & \ddots & \vdots \\ CC_{N1}^q & CC_{N2}^q & \cdots & CC_{Nm}^q \end{pmatrix}, \qquad (51)$$

where $q = \overline{1,n}$ – the number of the M_q method, CC_{ij}^q – the values of closeness coefficients for A_i $(i = \overline{1,N})$ alternatives, obtained according to the M_q method using the metric with the j $(j = \overline{1,m})$ number, while both CC_i^- coefficients of the form (42) and CC_i^+ ones of the form (43) can be chosen as closeness coefficients. Then the $D^{X,Y}$ matrix of deviations between the results of two methods X and Y $(X = \overline{1,n-1}, Y = \overline{2,n}, X \neq Y)$ is calculated according to the formula:

$$D^{X,Y} = $$
$$= \begin{pmatrix} |CC_{11}^X - CC_{11}^Y| & |CC_{12}^X - CC_{12}^Y| & \cdots & |CC_{1m}^X - CC_{1m}^Y| \\ |CC_{21}^X - CC_{21}^Y| & |CC_{22}^X - CC_{22}^Y| & \cdots & |CC_{2m}^X - CC_{2m}^Y| \\ \vdots & \vdots & \ddots & \vdots \\ |CC_{N1}^X - CC_{N1}^Y| & |CC_{N2}^X - CC_{N2}^Y| & \cdots & |CC_{Nm}^X - CC_{Nm}^Y| \end{pmatrix}, \qquad (52)$$

where CC_{ij}^x and CC_{ij}^y – the values of closeness coefficients in the corresponding methods X and Y, N – the number of alternatives, n – the number of methods, m – the number of metrics in each of applied methods.

Let M_1 be FT3FNs, M_2 – FT4FNs, M_3 – TFW3FNs and M_4 – TFW4FNs. After conducting an experiment with the considered methods with different j $(j = \overline{1,5})$ metrics for A_i $(i = \overline{1,3})$ alternatives, E^q $(q = 1,4)$ matrices of the form (51) shown in Fig. 10 were obtained.

$D^{X,Y}$ deviation matrices of the form (52) between the results of two methods X and Y $(X = \overline{1,3}, Y = \overline{2,4}, X \neq Y)$ are presented in Fig. 11.

	E^1	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
FT3FNs	A1	0,5764	0,5387	0,5789	0,6261	0,5970
	A2	0,4236	0,4613	0,4211	0,3739	0,4030
	A3	0,4524	0,4787	0,5217	0,4062	0,4354
	E^2	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
FT4FNs	A1	0,5419	0,5212	0,5714	0,6028	0,5574
	A2	0,4581	0,4788	0,4286	0,3972	0,4426
	A3	0,4914	0,5067	0,5556	0,4425	0,4792
	E^3	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
TFW3FNs	A1	0,6087	0,6055	0,5556	0,6631	0,6250
	A2	0,3282	0,2690	0,3000	0,3369	0,3363
	A3	0,5103	0,5656	0,6364	0,4571	0,4936
	E^4	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
TFW4FNs	A1	0,6119	0,6082	0,5556	0,6631	0,6282
	A2	0,3280	0,2689	0,3000	0,3369	0,3361
	A3	0,5083	0,5639	0,6364	0,4571	0,4914

Figure 10. Results of methods in the form of E^q matrix of the form (51)
Source: created by the authors

	$D^{1,2}$	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
|FT3FNs - FT4FNs|	A1	0,0345	0,0175	0,0075	0,0234	0,0396
	A2	0,0345	0,0175	0,0075	0,0234	0,0396
	A3	0,0389	0,0280	0,0338	0,0363	0,0438
	$D^{1,3}$	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
|FT3FNs - TFW3FNs|	A1	0,0324	0,0667	0,0234	0,0370	0,0280
	A2	0,0955	0,1922	0,1211	0,0370	0,0667
	A3	0,0579	0,0868	0,1146	0,0509	0,0582
	$D^{1,4}$	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
|FT3FNs - TFW4FNs|	A1	0,0355	0,0694	0,0234	0,0370	0,0312
	A2	0,0957	0,1923	0,1211	0,0370	0,0669
	A3	0,0559	0,0851	0,1146	0,0509	0,0559
	$D^{2,3}$	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
|FT4FNs - TFW3FNs|	A1	0,0669	0,0843	0,0159	0,0603	0,0676
	A2	0,1300	0,2098	0,1286	0,0603	0,1063
	A3	0,0190	0,0588	0,0808	0,0146	0,0144
	$D^{2,4}$	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
|FT4FNs - TFW4FNs|	A1	0,0700	0,0870	0,0159	0,0603	0,0707
	A2	0,1302	0,2099	0,1286	0,0603	0,1064
	A3	0,0170	0,0571	0,0808	0,0146	0,0122
	$D^{3,4}$	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
|TFW3FNs - TFW4FNs|	A1	0,003134289	0,002700502	0	0	0,00311769
	A2	0,000179553	9,22555E-05	0	0	0,000158067
	A3	0,002005483	0,001686576	0	0	0,002290157

Figure 11. $D^{X,Y}$ deviation matrices of the form (52) between the results of the methods
Source: created by the authors

As can be seen from Figure 11, the deviations are unevenly distributed, which means that different metrics respond differently to the same input data. The smallest deviation between closeness coefficients is obtained in the $D^{3,4}$ deviation matrix (Fig. 11), which indicates that there is no significant difference in the choice of the type of fuzzy numbers (triangular or trapezoidal FNs) for determining criteria weights. The largest deviation in $D^{3,4}$ is 0.0027 in the Manhattan metric. Also, insignificant deviations between closeness coefficients are observed in the $D^{1,2}$ deviation matrix (Fig. 11). Therefore, it can be concluded that there is no significant difference between the use of FTOPSIS using triangular and trapezoidal FNs. The largest deviation in $D^{1,2}$ is 0.0438 in the Minkowski metric. Relatively significant deviations between closeness coefficients are obtained in $D^{1,3}$, $D^{1,4}$, $D^{2,3}$ and $D^{2,4}$ deviation matrices. In particular, the largest deviations are observed when using the Manhattan metric (on average 0.2) for the above deviation matrices. This is due to the peculiarity of the calculation of this metric.

As a result of the experiments, the Chebyshev metric is determined to be the most stable metric (Fig. 11). The proposed approach for comparing the results of applying different methods makes it possible to determine which methods give similar or sharply different results. Using this approach, it is possible to analyse whether the scale of deviations is significant or whether it is within the limits of permissible error. Also, this approach to comparing the results of the use of different methods provides an opportunity to assess the quality of the experts' work, namely, how accurately the experts have carried out the fuzzification of real input data to the decision-making problem. If it is necessary to select a universal metric, then it is advisable to use metrics with minimal deviations (for example, the Minkowski or Chebyshev metric). If it is important to take into account individual maximum differences between FPIS and FNIS, then the Manhattan metric may be better. Methods that have minimal deviations between the results can be used alternatively, and methods that have significant deviations between the results require additional analysis in order to identify and explain the reasons for the obtained discrepancies. To compare the results of choosing the best alternative, a matrix of comparison of the ranks of alternatives is considered (Fig. 12).

FT3FNs	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
A1	1	1	1	1	1
A2	3	3	3	3	3
A3	2	2	2	2	2
FT4FNs	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
A1	1	1	1	1	1
A2	3	3	3	3	3
A3	2	2	2	2	2
TFW3FNs	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
A1	1	1	2	1	1
A2	3	3	3	3	3
A3	2	2	1	2	2
TFW4FNs	Euclidean	Manhattan	Hamming	Chebyshev	Minkowski
A1	1	1	2	1	1
A2	3	3	3	3	3
A3	2	2	1	2	2

Figure 12. Matrix of comparison of the ranks of alternatives

Source: created by the authors

According to the results of applying the FT3FNs, FT4FNs, TFW3FNs, TFW4FNs methods with different metrics (Fig. 12), it was found that ISO 31000:2018 (2018) (A_1 alternative) is the best option for the risk management standard for IT projects among the proposed alternatives. Within the framework of this study, a modified FTOPSIS method, which involves the use of adapted metrics that more correctly reflect the differences between fuzzy numbers, is proposed. In addition, the implemented information technology takes into account the results of group expertise, which provides increased accuracy in the process of multi-criteria selection. The conducted study is consistent with a number of previous works, which also emphasise the importance of metric selection in the context of fuzzy data. Thus, N. Rane & S. Choudhary (2023) and A. Makki & R. Abdulaal (2023) demonstrated the effectiveness of using FTOPSIS, but used the standard Euclidean distance.

Also, Y. Kustiyahningsih *et al.* (2024) demonstrated the use of FTOPSIS with the classical Euclidean metric, which, despite its prevalence, has a number of limitations in case of fuzzy or non-standardised data. H. Arman *et al.* (2022) critically analysed the feasibility of using the Euclidean metric in FTOPSIS, proving its conceptual inadequacy for comparing fuzzy numbers. In turn, H. Yin *et al.* (2020) proposed an alternative – relative Euclidean distance – that better takes into account the features of fuzzy data. The results obtained in P.Talukdar & P.Dutta (2019) and H.-S.Shyur & H.S. Shih (2024) confirm that the choice of metric can significantly affect the result of ranking alternatives. However, these studies do not focus on the fuzzy environment, which limits their relevance in the context of modern decision-making problems with a high level of uncertainty.

Based on the analysis of advantages and disadvantages of the analysed MADM methods, a modification of the FTOPSIS method is proposed using various metrics, in particular, the Euclidean metric, the Manhattan metric, the Chebyshev, Minkowski and Hamming metrics to estimate the closeness coefficient to fuzzy positive and fuzzy negative ideal solutions. Thus, the comparative analysis shows that the modification of the FTOPSIS method proposed in this work, taking into account the adapted metrics and group expertise, makes it possible to eliminate a number of shortcomings of previous approaches, providing a more accurate, flexible and substantiated assessment of alternatives in a multi-criteria environment. In the conducted scientific research:

- a modified FTOPSIS method is proposed using the results of group expertise and adaptation of popular metrics for triangular and trapezoidal fuzzy numbers, which ensures the preservation of metrics properties and allows to more accurately assess the distances between fuzzy ideal and anti-ideal solutions;
- an approach is proposed for determining criteria weights in the classical TOPSIS method through the use of linguistic assessments, which allows for more flexible consideration of uncertainty and subjectivity of expert assessments and is important for real decision-making scenarios;
- an approach is proposed for comparing the results of applying different methods, in particular FTOPSIS with the use of triangular and trapezoidal fuzzy numbers, TOPSIS with triangular and trapezoidal fuzzy numbers obtained using different metrics, which provides an opportunity to analyse the scale of deviations and assess the quality of experts' work.

CONCLUSIONS

The study analysed the most popular multi-criteria decision-making methods, in particular, such methods as TOPSIS, FTOPSIS and FAHP. Particular attention was paid to the analysis of different popular metrics for determining the distances between fuzzy positive and fuzzy negative ideal solutions in the FTOPSIS method. Based on the results of this analysis, a modified Fuzzy TOPSIS method for selecting an effective alternative in the multi-criteria decision-making problem based on different metrics and using the results of group expertise was proposed. On the example of a real problem of choosing a risk management standard in IT projects, it was demonstrated how the use of different metrics affects the accuracy and reliability of choosing an effective solution. An approach is proposed for comparing the results of applying different methods, in particular, the FTOPSIS methods with the use of triangular and trapezoidal fuzzy numbers, TOPSIS with triangular and trapezoidal fuzzy numbers for determining criteria weights obtained by different metrics for estimating the closeness coefficients to a fuzzy positive ideal solution (FPIS) and a fuzzy negative ideal solution (FNIS).

The results of the study open up new opportunities for applying the TOPSIS and FTOPSIS methods in various areas of decision-making, especially in conditions of multi-criteriality and uncertainty. The developed web-oriented application that implements the proposed modified Fuzzy TOPSIS method can also be used to solve multi-criteria decision-making problems in various fields of human activity under conditions of fuzzy data. Prospects for further research include improving methods for aggregating expert opinions, expanding the range of metrics used to increase the assessment accuracy, as well as adapting the proposed approach to dynamic decision-making conditions, in particular in real time and taking into account changes in criteria priorities. It is also relevant to study the possibilities of integrating the proposed approach with other approaches based on fuzzy logic, such as neural-fuzzy systems and hybrid models based on machine learning.

ACKNOWLEDGEMENTS

None.

FUNDING

None.

CONFLICT OF INTEREST

None.

REFERENCES

[1] Arman, H., Hadi-Vencheh, A., Kiani Mavi, R., Khodadadipour, M., & Jamshidi, A. (2022). Revisiting the interval and fuzzy TOPSIS methods: Is Euclidean distance a suitable tool to measure the differences between fuzzy numbers? *Complexity,* 2022, article number 7032662. doi: 10.1155/2022/7032662.

[2] Awasthi, A., Chauhan, S.S., & Goyal, S.K. (2011). A multi-criteria decision making approach for location planning for urban distribution centers under uncertainty. *Mathematical and Computer Modelling,* 53(1-2), 98-109. doi: 10.1016/j.mcm.2010.07.023.

[3] Black, P.E. (2019). *Manhattan distance*. Retrieved from https://xlinux.nist.gov/dads/HTML/manhattanDistance.html.

[4] Chen, C.T. (2000). Extension of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems,* 114(1), 1-9. doi: 10.1016/S0165-0114(97)00377-1.

[5] Chen, C.T., Lin, C.T., & Huang, S.F. (2006). A fuzzy approach for supplier evaluation and selection in supply chain management. *International Journal of Production Economics*, 102(2), 289-301. doi: 10.1016/j.ijpe.2005.03.009.
[6] Decisioner. (n.d.). Retrieved from https://dss.knsa.chdtu.edu.ua/.
[7] Deza, M.M., & Deza, E. (2016). *Encyclopedia of distances*. Berlin: Springer. doi: 10.1007/978-3-662-52844-0.
[8] Han, F., Alkhawaji, R., & Shafieezadeh, M. (2024). Evaluating sustainable water management strategies using TOPSIS and fuzzy TOPSIS methods. *Applied Water Science,* 15, article number 4. doi: 10.1007/s13201-024-02336-7.
[9] Hwang, C.L., & Yoon, K.P. (1981). *Multiple attribute decision making. Methods and applications. A state-of-the-art survey*. Berlin: Springer. doi: 10.1007/978-3-642-48318-9.
[10] ISO 31000:2018. (2018). *Risk management – guidelines*. Retrieved from https://www.iso.org/iso-31000-risk-management.html.
[11] Kustiyahningsih, Y., Rahmanita, E., Khusnul Khotimah, B., & Purnama, J. (2024). Type-2 fuzzy ANP and TOPSIS methods based on trapezoid fuzzy number with a new metric. *International Journal of Advances in Intelligent Informatics,* 10(2), article number 239. doi: 10.26555/ijain.v10i2.1285.
[12] Makki, A., & Abdulaal, R. (2023). A hybrid MCDM approach based on fuzzy MEREC-G and fuzzy RATMI. *Mathematics*, 11(17), article number 3773. doi: 10.3390/math11173773.
[13] Maksymov, A. (2025). Analysis of risk management standards and their application in IT projects. *Management of Development of Complex Systems*, 61, 66-75. doi: 10.32347/2412-9933.2025.61.66-75.
[14] National Institute of Standards and Technology Special Publication 800-53. (2020). Retrieved from https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-53r5.pdf.
[15] PMBOK guide. (2021). Retrieved from https://www.pmi.org/standards/pmbok.
[16] Rane, N., & Choudhary, S. (2023). Fuzzy AHP and fuzzy TOPSIS as an effective and powerful multi-criteria decision-making (MCDM) method for subjective judgments in selection process. *International Research Journal of Modernization in Engineering Technology and Science,* 5, 3786-3799. doi: 10.56726/IRJMETS36629.
[17] Rudin, W. (1987). *Real and complex analysis*. Singapore: McGraw-Hill.
[18] Shyur, H.-J., & Shih, H.-S. (2024). Resolving rank reversal in TOPSIS: A comprehensive analysis of distance metrics and normalization methods. *Informatica*, 35(4), 837-858. doi: 10.15388/24-INFOR576.
[19] Talukdar, P., & Dutta, P. (2019). A comparative study of TOPSIS method via different distance measure. *International Journal of Research in Advent Technology,* 7(5), 118-126. doi: 10.32622/ijrat.75201937.
[20] Yin, H., Li, X., & Gao, Y. (2020). Relative Euclidean distance with application to TOPSIS and estimation performance ranking. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 52(2), 1052-1064. doi: 10.1109/TSMC.2020.3017814.

Інформаційна технологія розв'язання задачі багатокритеріального прийняття рішень модифікованим методом Fuzzy TOPSIS

Антон Максимов

Аспірант
Черкаський державний технологічний університет
18006, бульв. Шевченка, 460, м. Черкаси, Україна
https://orcid.org/0009-0002-0411-8164

Юрій Триус

Доктор педагогічних наук, кандидат фізико-математичних наук, професор
Черкаський державний технологічний університет
18006, бульв. Шевченка, 460, м. Черкаси, Україна
https://orcid.org/0000-0002-0739-2065

Анотація. Актуальність теми дослідження зумовлена потребою ефективного розв'язання задач багатокритеріального прийняття рішень в умовах нечіткої інформації. У зв'язку з цим важливою проблемою є створення інформаційних технологій, які б надавали можливість користувачу обирати і застосовувати найбільш ефективні методи багатокритеріального прийняття рішень в умовах нечіткої інформації. Метою дослідження була розробка інформаційної технології розв'язання задачі багатокритеріального прийняття рішень модифікованим методом Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) на основі використання різних метрик і результатів групової експертизи, що підвищує достовірність отриманих рішень. У межах дослідження проведено аналіз найпопулярніших методів багатокритеріального прийняття рішень, зокрема методів, що використовують апарат нечітких множин. У статті проведено аналіз різних популярних метрик для оцінки відстаней між нечітким позитивним ідеальним розв'язком та нечітким негативним ідеальним розв'язком в методі FTOPSIS. Запропоновано методику для порівняння результатів застосування різних методів, зокрема FTOPSIS із застосуванням трикутних та трапецієвидних нечітких чисел, TOPSIS з трикутними та трапецієвидними нечіткими числами для визначення ваг критеріїв, що надає можливість проаналізувати масштаби відхилень між одержаними результатами та оцінити якість роботи експертів. Отримані результати розширюють можливості використання методів TOPSIS і FTOPSIS для прийняття рішень в умовах багатокритеріальності та невизначеності. Як практичне застосування розробленої інформаційної технології і модифікованого методу FTOPSIS у статті розв'язується задача вибору найкращого з популярних стандартів управління ризиками в IT-проєктах. Це дозволить підвищити ефективність ризик-менеджменту в умовах невизначеності та неповноти інформації, підвищити обґрунтованість прийнятих рішень, а також адаптувати процес управління ризиками до специфічних умов кожного окремого IT-проєкту

Ключові слова: MCDM; MADM; нечіткі множини; FTOPSIS; IT-проєкти; управління ризиками; стандарти управління ризиками